MATHEMATIC

PUNJAB BOARD NOTES

9TH CLASS

Presented by:

Urdu Books Whatsapp Group

STUDY GROUP

0333-8033313
راؤ ایاز

0343-7008883
پاکستان زندہ باد

0306-7163117
محمد سلمان سلیم

Exercise 1.1

Q1. Find the order of the following matrices.

A = $\begin{bmatrix} 2 & 3 \\ -5 & 6 \end{bmatrix}$, **B** = $\begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$, **C** = $\begin{bmatrix} 2 & 4 \end{bmatrix}$,

D = $\begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$, **E** = $\begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$, **F** = $\begin{bmatrix} 2 \end{bmatrix}$,

$G = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$, **H** = $\begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 6 \end{bmatrix}$

Solution:

Order of the Matrix:

The number of rows and columns in a Matrix specifies its order.

Ans. (i) Matrix A has two rows and two columns
So, its order = number of rows x number of columns = 2-by-2.

Ans. (ii) Matrix B has two rows and two columns
So, its order = number of rows x number of columns = 2-by-2.

Ans. (iii) Matrix C has one row and two columns
So, its order = number of rows x number of columns = 1-by-2.

Ans. (iv) Matrix D has three rows and one column
So, its order = number of rows x number of columns = 3-by-1.

Ans. (v) Matrix E has three rows and two columns

So, its order = number of rows x number of columns = 3-by-2.

Ans. (vi) Matrix F has one row and one column

So, its order = number of rows x number of columns = 1-by-1.

Ans. (vii) Matrix G has three rows and three columns

So, its order = number of rows x number of columns = 3-by-3.

Ans. (viii) Matrix A has two rows and three columns

So, its order = number of rows x number of columns = 2-by-3.

Q2. Which of the following matrices are equal?

A = $\begin{bmatrix}3\end{bmatrix}$, **B** = $\begin{bmatrix}3 & 5\end{bmatrix}$, **C** = $\begin{bmatrix}5-2\end{bmatrix}$,

D = $\begin{bmatrix}5 & 3\end{bmatrix}$, **E** = $\begin{bmatrix}4 & 0\\ 6 & 2\end{bmatrix}$, **F** = $\begin{bmatrix}2\\ 6\end{bmatrix}$,

G = $\begin{bmatrix}3-1\\ 3+3\end{bmatrix}$, **H** = $\begin{bmatrix}4 & 0\\ 6 & 2\end{bmatrix}$, **I** = $\begin{bmatrix}3 & 3+2\end{bmatrix}$,

J = $\begin{bmatrix}2+2 & 2-2\\ 2+4 & 2+0\end{bmatrix}$

Solution:

Solving C

$$C = \begin{bmatrix}5-2\end{bmatrix}$$

$$C = \begin{bmatrix}3\end{bmatrix}$$

Solving G

$$G = \begin{bmatrix}3-1\\ 3+3\end{bmatrix}$$

$$G = \begin{bmatrix}2\\ 6\end{bmatrix}$$

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السلام علیکم ورحمۃ اللہ وبرکاتہ:

معزز ممبران: آپ کا وٹس ایپ گروپ ایڈ من "**اردو بکس**" آپ سے مخاطب ہے۔

آپ تمام ممبران سے گزارش ہے کہ:

- گروپ میں صرف PDF کتب پوسٹ کی جاتی ہیں لہذا کتب کے متعلق اپنے کمنٹس / ریویوز ضرور دیں۔ گروپ میں بغیر ایڈ من کی اجازت کے کسی بھی قسم کی (اسلامی وغیر اسلامی، اخلاقی، تحریری) پوسٹ کرنا سختی سے منع ہے۔
- گروپ میں معزز، پڑھے لکھے، سلجھے ہوئے ممبرز موجود ہیں اخلاقیات کی پابندی کریں اور گروپ رولز کو فالو کریں بصورت دیگر معزز ممبرز کی بہتری کی خاطر ریموو کر دیا جائے گا۔
- کوئی بھی ممبر کسی بھی ممبر کو انباکس میں میسج، مس کال، کال نہیں کرے گا۔ رپورٹ پر فوری ریموو کر کے کاروائی عمل میں لائے جائے گی۔
- ہمارے کسی بھی گروپ میں سیاسی و فرقہ واریت کی بحث کی قطعاً کوئی گنجائش نہیں ہے۔
- اگر کسی کو بھی گروپ کے متعلق کسی قسم کی شکایت یا تجویز کی صورت میں ایڈ من سے رابطہ کیجئے۔
- سب سے اہم بات:

گروپ میں کسی بھی قادیانی، مرزائی، احمدی، گستاخِ رسول، گستاخِ امہات المؤمنین، گستاخِ صحابہ و خلفائے راشدین حضرت ابو بکر صدیق، حضرت عمر فاروق، حضرت عثمان غنی، حضرت علی المرتضیٰ، حضرت حسنین کریمین رضوان اللہ تعالیٰ اجمعین، گستاخ اہلبیت یا ایسے غیر مسلم جو اسلام اور پاکستان کے خلاف پراپیگنڈا میں مصروف ہیں یا ان کے روحانی وذہنی سپورٹرز کے لئے کوئی گنجائش نہیں ہے لہذا ایسے اشخاص بالکل بھی گروپ جوائن کرنے کی زحمت نہ کریں۔ معلوم ہونے پر فوراً ریموو کر دیا جائے گا۔

- تمام کتب انٹرنیٹ سے تلاش / ڈاؤنلوڈ کر کے **فری آف کاسٹ** وٹس ایپ گروپ میں شیئر کی جاتی ہیں۔ جو کتاب نہیں ملتی اس کے لئے معذرت کر لی جاتی ہے۔ جس میں محنت بھی صَرف ہوتی ہے لیکن ہمیں آپ سے صرف دعاؤں کی درخواست ہے۔
- عمران سیریز کے شوقین کیلئے علیحدہ سے عمران سیریز گروپ موجود ہے۔
- **لیڈیز کے لئے الگ گروپ کی سہولت موجود ہے جس کے لئے ویریفکیشن ضروری ہے۔**
- اردو کتب / عمران سیریز یا سٹڈی گروپ میں ایڈ ہونے کے لئے ایڈ من سے وٹس ایپ پر بذریعہ میسج رابطہ کریں اور جواب کا انتظار فرمائیں۔ برائے مہربانی اخلاقیات کا خیال رکھتے ہوئے موبائل پر کال یا ایم ایس کرنے کی کوشش ہر گز نہ کریں۔ ورنہ گروپس سے تو ریموو کیا ہی جائے گا بلاک بھی کیا جائے گا۔

نوٹ: ہمارے کسی گروپ کی کوئی فیس نہیں ہے۔ سب فی سبیل اللہ ہے

محمد سلمان سلیم	پاکستان زندہ باد	راؤ ایاز
0306-7163117	0343-7008883	0333-8033313

پاکستان زندہ باد　　　　　　پاکستان پائندہ باد

اللہ تبارک تعالیٰ ہم سب کا حامی وناصر ہو

Solving I

$$I = \begin{bmatrix} 3 & 3+2 \end{bmatrix}$$

$$I = \begin{bmatrix} 3 & 5 \end{bmatrix}$$

Solving J

$$J = \begin{bmatrix} 2+2 & 2-2 \\ 2+4 & 2+0 \end{bmatrix}$$

$$J = \begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$$

Now

Matrices are said to be equal if

(i) They are of same order
(ii) Their corresponding values are equal

So, according to this definition

Ans. **(a)** Matrices A and C are equal, A = C.

(b) Matrices B and I are equal, B = I.

(c) Matrices E, H and J are equal, E = H = J.

(d) Matrices F and G are equal, F = G.

Q3. Find the values of a, b, c, and d which satisfy the matrix equation.

$$\begin{bmatrix} a+c & a+2b \\ c-1 & 4d-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2d \end{bmatrix}$$

Solution:

As, $\begin{bmatrix} a+c & a+2b \\ c-1 & 4d-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2d \end{bmatrix}$

Exercise 1.2

Q1. From the following matrices, identify unit matrices, row matrices, column matrices and null matrices.

A = $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, **B =** $\begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$, **C =** $\begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$,

D = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, **E =** $\begin{bmatrix} 0 \end{bmatrix}$, **F =** $\begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$

Solution:

Matrix A is a null matrix (because all its entries are zero)

Matrix B is a row matrix (because it has only one row).

Matrix C is a column matrix (because it has only one column).

Matrix D is a unit matrix (because its diagonal entries are 1 and non-diagonal entries are 0).

Matrix E is a null matrix (because all its entries are 0).

Matrix F is a column matrix (because it has only one column).

Q2. From the following matrices, identify

(a) Square matrices **(b) Rectangular matrices**

(c) Row matrices **(d) Column matrices**

(e) Identity matrices **(f) Null matrices**

(i) $\begin{bmatrix} -8 & 2 & 7 \\ 12 & 0 & 4 \end{bmatrix}$, **(ii)** $\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$, **(iii)** $\begin{bmatrix} 6 & -4 \\ 3 & -2 \end{bmatrix}$,

(iv) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, **(v)** $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$, **(vi)** $\begin{bmatrix} 3 & 10 & -1 \end{bmatrix}$,

$(\text{vii})\begin{bmatrix}1\\0\\0\end{bmatrix}$, **(viii)** $\begin{bmatrix}1 & 2 & 3\\-1 & 2 & 0\\0 & 0 & 1\end{bmatrix}$ **(ix)** $\begin{bmatrix}0 & 0\\0 & 0\\0 & 0\end{bmatrix}$

Solution:

(a) (iii), (iv) and (viii) are square matrices because the number of rows are equal to number of columns.

(b) (i), (ii), (v), (vi) (vii), (ix) are rectangular matrices because their rows and columns are not equal.

(c) (vi) is a row matrix because it has only one row.

(d) (ii) and (vii) are column matrices because they have only one column.

(e) (iv) is an identity matrix as well because its diagonal elements are "1" and all non-diagonal elements are zero.

(f) (ix) is a null matrix because its each entry is zero.

Q3. From the following matrices identify diagonal, scalar and unit (identity) matrices.

A = $\begin{bmatrix}4 & 0\\0 & 4\end{bmatrix}$, **B** = $\begin{bmatrix}2 & 0\\0 & -1\end{bmatrix}$, **C** = $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$,

D = $\begin{bmatrix}3 & 0\\0 & 0\end{bmatrix}$, **E** = $\begin{bmatrix}5-3 & 0\\0 & 1+1\end{bmatrix}$

Solution:

Matrix A is a scalar matrix (because its diagonal entries are same).

Matrix B is a diagonal matrix (because its diagonal entries are non-zero and non-diagonal entities are zero).

Matrix C is an identity matrix (because its diagonal entries are 1).

Matrix D is a diagonal matrix (because its one diagonal entry is non-zero and non-diagonal entities are zero).

Matrix E is a scalar matrix (because its diagonal entries are same).

$$E = \begin{bmatrix}5-3 & 0\\0 & 1+1\end{bmatrix}$$

$$E = \begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}$$

(Note that all the matrices A, B, C, D and E are diagonal matrices because they have at least one non-zero diagonal entry and all their non-diagonal entries are zero.

Scalar and unit matrices are further types of diagonal matrices.)

Q4. Find negative of matrices A, B, C, D and E when:

A = $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, **B =** $\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$, **C =** $\begin{bmatrix} 2 & 6 \\ 3 & 2 \end{bmatrix}$,

D = $\begin{bmatrix} -3 & 2 \\ -4 & 5 \end{bmatrix}$, **E =** $\begin{bmatrix} 1 & -5 \\ 2 & 3 \end{bmatrix}$

Solution:

Negative of a matrix is obtained by inverting (changing) the signs of all its entries.

So,

(i) $-A = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ **(ii)** $-B = \begin{bmatrix} -3 & 1 \\ -2 & -1 \end{bmatrix}$

(iii) $-C = \begin{bmatrix} -2 & -6 \\ -3 & -2 \end{bmatrix}$ **(iv)** $-D = \begin{bmatrix} 3 & -2 \\ 4 & -5 \end{bmatrix}$

(v) $-E = \begin{bmatrix} -1 & 5 \\ -2 & -3 \end{bmatrix}$

Q5. Find the transpose of each of the following matrices:

A = $\begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$, **B =** $\begin{bmatrix} 5 & 1 & -6 \end{bmatrix}$, **C =** $\begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}$,

D = $\begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$, **E =** $\begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}$, **F =** $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Solution:

Exercise 1.3

Q1. Which of the following matrices are conformable for addition?

A = $\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$, **B =** $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$, **C =** $\begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 1 & -2 \end{bmatrix}$,

D = $\begin{bmatrix} 2+1 \\ 3 \end{bmatrix}$, **E =** $\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, **F =** $\begin{bmatrix} 3 & 2 \\ 1+1 & -4 \\ 3+2 & 2+1 \end{bmatrix}$

Solution:

Solving D

$$D = \begin{bmatrix} 2+1 \\ 3 \end{bmatrix}$$

$$D = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

Solving F

$$F = \begin{bmatrix} 3 & 2 \\ 1+1 & -4 \\ 3+2 & 2+1 \end{bmatrix}$$

$$F = \begin{bmatrix} 3 & 2 \\ 2 & -4 \\ 5 & 3 \end{bmatrix}$$

Now

Matrices of same order are conformable for addition.

So, according to this definition;

(i) Matrices A and E are conformable for addition (because both have order 2-by-2).

(ii) Matrices B and D are conformable for addition (because both have order 1-by-1).
(iii) Matrices C and F are conformable for addition (because both have order 3-by-2).

Q2. Find the additive inverse of following matrices.

A = $\begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix}$, **B =** $\begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & -2 & 1 \end{bmatrix}$, **C =** $\begin{bmatrix} 4 \\ -2 \end{bmatrix}$,

D = $\begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix}$, **E =** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, **F =** $\begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$

Solution:

The additive inverse of a matrix is obtained by changing the sign of each entry.

So, according to the definition;

(i) Additive inverse of A = - A = $\begin{bmatrix} -2 & -4 \\ 2 & -1 \end{bmatrix}$

(ii) Additive inverse of B = - B = $\begin{bmatrix} -1 & 0 & 1 \\ -2 & 1 & -3 \\ -3 & 2 & -1 \end{bmatrix}$

(iii) Additive inverse of C = - C = $\begin{bmatrix} -4 \\ 2 \end{bmatrix}$

(iv) Additive inverse of D = - D = $\begin{bmatrix} -1 & 0 \\ 3 & 2 \\ -2 & -1 \end{bmatrix}$

(v) Additive inverse of E = - E = $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

(vi) Additive inverse of F = - F = $\begin{bmatrix} -\sqrt{3} & -1 \\ 1 & -\sqrt{2} \end{bmatrix}$

Q3. **If**

A = $\begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$, **B =** $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$, **C =** $\begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$, **D =** $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$,

then find:

(i) $A+\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ **(ii)** $B+\begin{bmatrix} -2 \\ 3 \end{bmatrix}$ **(iii)** $C+\begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$

(iv) $D+\begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ **(v)** **2A** **(vi)** **(-1) B**

(vii) **(-2) C** **(viii)** **3D** **(ix)** **3C**

Solution:

(i) **A +** $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$$= A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1+1 & 2+1 \\ 2+1 & 1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 3 \\ 3 & 2 \end{bmatrix}$$

So, $A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ 3 & 2 \end{bmatrix}$

(ii) **B +** $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$$= \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

Exercise 1.4

Q1. Which of the following product of matrices is conformable for multiplication?

(i) $\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$ **(iv)** $\begin{bmatrix} 1 & 2 \\ 0 & -1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$

(v) $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ -2 & 3 \end{bmatrix}$

Solution:

Two matrices are conformable for multiplication if the numbers of columns of first matrix are equal to number of rows of second matrix.

So, according to this definition:

(i) is conformable for multiplication (because the first matrix has two columns and second matrix has same number of rows).

(ii) is conformable for multiplication (because the first matrix has two columns and second matrix has same number of rows).

(iii) is not conformable for multiplication (because the first matrix has just one column and second matrix has two rows).

(iv) is conformable for multiplication (because the first matrix has just two columns and second matrix has the same number of rows).

(v) is conformable for multiplication (because the first matrix has three columns and second matrix has same number of rows).

Q2. **If A =** $\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$, **B =** $\begin{bmatrix} 6 \\ 5 \end{bmatrix}$,

find (i) AB (ii) BA (if possible)

Solution:

(i) AB

$$= \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times6+0\times5 \\ (-1)\times6+2\times5 \end{bmatrix}$$

$$= \begin{bmatrix} 18+0 \\ -6+10 \end{bmatrix}$$

$$= \begin{bmatrix} 18 \\ 4 \end{bmatrix}$$

So, AB $= \begin{bmatrix} 18 \\ 4 \end{bmatrix}$

(ii) BA

BA is not possible (because number of columns of B is

Not equal to number of rows of A)

Q3. **Find the following products**

(i) $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \end{bmatrix}$

(iii) $\begin{bmatrix}-3 & 0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$ **(iv)** $\begin{bmatrix}6 & -0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$

(v) $\begin{bmatrix}1 & 2\\-3 & 0\\6 & -1\end{bmatrix}\begin{bmatrix}4 & 5\\0 & -4\end{bmatrix}$ **(vi)** $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$

Solution:

(i) $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$

$= \begin{bmatrix}1\times4+2\times0\end{bmatrix}$

$= \begin{bmatrix}4+0\end{bmatrix}$

$= \begin{bmatrix}4\end{bmatrix}$

So, $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix} = \begin{bmatrix}4\end{bmatrix}$

(ii) $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}5\\-4\end{bmatrix}$

$= \begin{bmatrix}1\times5+2\times(-4)\end{bmatrix}$

$= \begin{bmatrix}5-8\end{bmatrix}$

$= \begin{bmatrix}-3\end{bmatrix}$

So, $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}5\\-4\end{bmatrix} = \begin{bmatrix}-3\end{bmatrix}$

(iii) $\begin{bmatrix}-3 & 0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$

Exercise 1.5

Q1. Find the determinant of the following matrices.

(i) **A =** $\begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$ **(ii)** **B =** $\begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$

(iii) **C =** $\begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$ **(iv)** **D =** $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

Solution:

(i) **A =** $\begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$

Determinant of matrix Ais calculated as:

$|A|$ = det A = $\begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$ = (-1) x 0 - 2 x 1

$|A|$ **= 0 – 2 = – 2**

(ii) **B =** $\begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$

Determinant of matrix B is calculated as:

$|B|$ = det B = $\begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$ = 1 x (-2) - 3 x 2

$|B|$ **= – 2 – 6 = – 8**

(iii) **C =** $\begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$

Determinant of matrix C is calculated as:

$|C|$ = det C = $\begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$ = 3 x 2 - 2 x 3

$|C|$ **= 6 – 6 = 0**

(iv) **D =** $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

Determinant of matrix D is calculated as:

$|D|$ = det D = $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ = 3 x 4 - 2 x 1

$|D|$ **= 12 – 2 = 10**

Q2. Find which of the following matrices are singular or non-singular?

(i) **A =** $\begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$ **(ii)** **B =** $\begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$

(iii) **C =** $\begin{bmatrix} 7 & -9 \\ 3 & 5 \end{bmatrix}$ **(iv)** **D =** $\begin{bmatrix} 5 & -10 \\ -2 & 4 \end{bmatrix}$

Solution:

A matric is said to be singular if its determinant is equal to zero. i.e., $|A| = 0$

(i) **A =** $\begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$

Determinant of matrix A is calculated as:

$|A|$ = det A = $\begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$ = 3 x 4 - 2 x 6

$|\mathbf{A}|$ **= 12 – 12 = 0**

As, determinant of A is equal to zero so, A is a singular matrix.

(ii) **B =** $\begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$

Determinant of matrix B is calculated as:

$|B|$ = det B = $\begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$ = 4 x 2 - 1x 3

$|\mathbf{B}|$ **= 8 - 3 = 5 $\neq$ 0**

As, determinant of B is not equal to zero so, B is a not a singular matrix.

(iii) **C =** $\begin{bmatrix} 7 & -9 \\ 3 & 5 \end{bmatrix}$

Determinant of matrix C is calculated as:

$|C|$ = det C = $\begin{bmatrix} 7 & -9 \\ 3 & 5 \end{bmatrix}$ = 7 x 5 – 3 x (-9)

$|\mathbf{C}|$ **= 35 + 27 = 62 $\neq$ 0**

As, determinant of C is not equal to zero so, C is a not a singular matrix.

(iv) **D =** $\begin{bmatrix} 5 & -10 \\ -2 & 4 \end{bmatrix}$

Determinant of matrix D is calculated as:

$|D|$ = det D = $\begin{bmatrix} 5 & -10 \\ -2 & 4 \end{bmatrix}$ = 5 x 4 – (-10) x (-2)

$|\mathbf{D}|$ **= 20 – 20 = 0**

Exercise 1.6

Q1. Use matrices, if possible, to solve the following systems of linear equations by:

(i) the matrix inversion method

(ii) the Cramer's rule

(i)	**$2x - 2y = 4$**	**(ii)**	**$2x + y = 3$**
	$3x + 2y = 6$		**$6x + 5y = 1$**
(iii)	**$4x + 2y = 8$**	**(iv)**	**$3x - 2y = -6$**
	$3x - y = -1$		**$5x - 2y = -10$**
(v)	**$3x - 2y = 4$**	**(vi)**	**$4x + y = 9$**
	$-6x + 4y = 7$		**$-3x - y = -5$**
(vii)	**$2x - 2y = 4$**	**(viii)**	**$3x - 4y = 4$**
	$-5x - 2y = -10$		**$x + 2y = 8$**

(i) Solution by Matrix Inversion Method:

(i) **$2x - 2y = 4$**

$3x + 2y = 6$

Step 1

$$\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

Step 2

The coefficient matrix M = $\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$ is non-singular because

det M = $\begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix}$ = $2\times2-3\times(-2)$ = 4 + 6 = 10 $\neq$ 0

Step 3

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1}\begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|M|} Adj\,M \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 2\times4+2\times6 \\ -3\times4+2\times6 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 8+12 \\ -12+12 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 20 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$\therefore \quad x=2\ ,\ \ y=0$

(ii) 2x + y = 3

6x + 5y = 1

Step 1

$$\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Step 2

The coefficient matrix M = $\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}$ is non-singular because

det M = $\begin{vmatrix} 2 & 1 \\ 6 & 5 \end{vmatrix}$ = $2\times5-1\times6$ = 10 - 6 = 4 $\neq$ 0

Step 3

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|M|} Adj\, M \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 5\times3+(-1)\times1 \\ -6\times3+2\times1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 14 \\ -16 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{7}{2} \\ -4 \end{bmatrix}$$

$\therefore \quad x = \frac{7}{2}, \; y = -4$

(iii) 4x + 2y = 8

3x − y = − 1

Step 1

$$\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

Step 2

The coefficient matrix M = $\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$ is non-singular because

det M = $\begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix}$ = $4\times(-1)-2\times3$ = -4 - 6 = - 10 $\neq$ 0

Step 3

Review Exercise

Q1. Select the correct answer in each of the following.

(i) The order of matrix $\begin{bmatrix} 2 & 1 \end{bmatrix}$ is …………

(a) 2-by-1 (b) 1-by-2

(c) 1-by-1 (d) 2-by-2

(ii) $\begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$ is called ………… matrix.

(a) zero (b) unit

(b) scalar (d) singular

(iii) Which is order of square matrix…………

(a) 2-by-2 (b) 1-by-2

(c) 2-by-1 (d) 3-by-2

(iv) Order of transpose of $\begin{bmatrix} 2 & 1 \\ 0 & 1 \\ 3 & 2 \end{bmatrix}$ …………

(a) 3-by-2 (b) 2-by-3

(c) 1-by-3 (d) 3-by-1

(v) Adjoint of $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$ is …………

(a) $\begin{bmatrix} -1 & -2 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 2 \\ 0 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$

(vi) Product of $\begin{bmatrix} x & y \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ is............

(a) $[2x+y]$ (b) $[x-2y]$

c) $[2x-y]$ (d) $[x+2y]$

(vii) If $\begin{vmatrix} 2 & 6 \\ 3 & x \end{vmatrix}$ = 0, then x is equal to…a = ...

(a) 9 (b) - 6

(c) 6 (d) - 9

(viii) If $X+\begin{bmatrix} -1 & -2 \\ 0 & -1 \end{bmatrix}$ = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then X is equal to............

(a) $\begin{bmatrix} 2 & 2 \\ 2 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 2 \\ 2 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$

Answers

(i) b	(ii) c	(iii)a	(iv) b
(v) a	(vi) c	(vii) a	(viii) d

Q2. Complete the following:

(i) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is called ……….. matrix.

(ii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is called ……….. matrix.

(iii) Additive inverse of $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$ is ………..

(iv) In matrix multiplication, in general, AB ……….. BA.

(v) Matrix A + B may be found if order of A and B is ………..

(vi) A matrix is called ………. matrix if number of rows and columns are equal.

Answers:

(i) Null **(ii) Unit** **(iii)** $\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$

(iv) $\neq$ **(v) Same** **(vi) Square**

Q3. **If** $\begin{bmatrix} a+3 & 4 \\ 6 & b-1 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$**, then find a and b.**

Solution:

By comparing the corresponding elements, we get

$a+3=-3$

$a=-3-3=-6$

$a=-6$ Answer

and $b-1=2$

$b=2+1$

Exercise 1.3

Q.1 Which of the following are conformable for addition?

$$A=\begin{bmatrix}2 & 1\\ -1 & 3\end{bmatrix}, \quad B=\begin{bmatrix}3\\ 1\end{bmatrix}$$

$$C=\begin{bmatrix}1 & 0\\ 2 & -1\\ 1 & -2\end{bmatrix}, \quad D=\begin{bmatrix}2+1\\ 3\end{bmatrix}$$

$$E=\begin{bmatrix}-1 & 0\\ 1 & 2\end{bmatrix}, \quad F=\begin{bmatrix}3 & 2\\ 1+1 & -4\\ 3+2 & 2+1\end{bmatrix}$$

Solution:
In the above matrices following matrices are suitable for addition.

(i) A and E are conformable for addition because their order is same and both are square matrix.

(ii) B and D are conformable for addition because the order is same i.e. they have two rows and 1 Columns and both are rectangular matrices.

(iii) C and F are conformable for addition because their order is same i.e. they have three 3 rows and 2 columns and they are a rectangular matrix.

Q.2 Find the additive inverse of the following matrices:

(1) $A=\begin{bmatrix}2 & 4\\ -2 & 1\end{bmatrix}$

Solution:
Additive inverse of a matrix is negative matrix.

$A=\begin{bmatrix}2 & 4\\ -2 & 1\end{bmatrix}$ is

$$-A=-\begin{bmatrix}2 & 4\\ -2 & 1\end{bmatrix}=\begin{bmatrix}(-1)\times 2 & (-1)4\\ (-1)(-2) & (-1)1\end{bmatrix}$$

$$-A=\begin{bmatrix}-2 & -4\\ 2 & -1\end{bmatrix}$$

(2) $B=\begin{bmatrix}1 & 0 & -1\\ 2 & -1 & 3\\ 3 & -2 & 1\end{bmatrix}$

Solution: $B=\begin{bmatrix}1 & 0 & -1\\ 2 & -1 & 3\\ 3 & -2 & 1\end{bmatrix}$

Its additive inverse is

$$-B=-\begin{bmatrix}+1 & 0 & -1\\ +2 & -1 & 3\\ +3 & -2 & 1\end{bmatrix}$$

$$-B=\begin{bmatrix}-1 & 0 & 1\\ -2 & 1 & -3\\ -3 & 2 & -1\end{bmatrix}$$

(3) $C = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

Solution: $C = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

$$-C = -\begin{bmatrix} 4 \\ -2 \end{bmatrix} = \begin{bmatrix} -1\times 4 \\ -1\times -2 \end{bmatrix}$$

The additive inverse is

$$-C = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$$

(4) $D = \begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix}$

Solution: $D = \begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix}$

The additive inverse is

$$-D = -\begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -1\times 1 & -1\times 0 \\ -1\times -3 & -1\times -2 \\ -1\times 2 & -1\times 1 \end{bmatrix}$$

$$-D = \begin{bmatrix} -1 & 0 \\ 3 & 2 \\ -2 & -1 \end{bmatrix}$$

(5) $E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Solution: $E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

The additive inverse of the given matrix is:

$$-E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1\times 1 & -1\times 0 \\ -1\times 0 & -1\times 1 \end{bmatrix}$$

$$-E = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

(6) $F = \begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$

Solution: $F = \begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$

Its additive inverse is

$$-F = -\begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} -1\times\sqrt{3} & -1\times 1 \\ -1\times -1 & -1\times\sqrt{2} \end{bmatrix}$$

$$-F = \begin{bmatrix} -\sqrt{3} & -1 \\ 1 & -\sqrt{2} \end{bmatrix}$$

Q.3 **If** $A = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ -1 \end{bmatrix},$

$C = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix},\ D = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix},$

then find.

(i) $A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Solution: $A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

As $A = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$

So, $A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

The order of matrix A and the given matrix order is same. So, they can be added easily.

$$= \begin{bmatrix} -1+1 & 2+1 \\ 2+1 & 1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 3 \\ 3 & 2 \end{bmatrix}$$

(ii) $B + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

Solution: $B + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

As $B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

So, $B + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

The order of both above matrices are same, so, they can be easily added.

$= \begin{bmatrix} 1+(-2) \\ -1+3 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -1 & +3 \end{bmatrix}$

$= \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

(iii) $C + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$

Solution: $C + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$

As $C = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$

So, $C + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1 & -1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$

Their orders are same so they can added

$= \begin{bmatrix} 1+(-2) & -1+(1) & 2+3 \end{bmatrix}$

$= \begin{bmatrix} 1-2 & -1+1 & 5 \end{bmatrix}$

$= \begin{bmatrix} -1 & 0 & 5 \end{bmatrix}$

(iv) $D + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

Solution: $D + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

As $D = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$

So, $D + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

Their orders are same. So, they can be added.

$= \begin{bmatrix} 1+0 & 2+1 & 3+0 \\ -1+2 & 0+0 & 2+1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 3 & 3 \\ 1 & 0 & 3 \end{bmatrix}$

(v) 2*A*

Solution: $2A$

As $A = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$

So, $2A$

$= (2) \times \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2(-1) & 2\times 2 \\ 2\times 2 & 2\times 1 \end{bmatrix}$

$= \begin{bmatrix} -2 & 4 \\ 4 & 2 \end{bmatrix}$

(vi) $(-1)B$

Solution: $(-1)B$

As $B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

So, $(-1)B$

$= (-1) \times \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$= \begin{bmatrix} (-1)\times 1 \\ (-1)\times(-1) \end{bmatrix}$

$= \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

(vii) $(-2)C$

Solution: $(-2)C$

As $C = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$

So, (-2) C

$= (-2) \times \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} (-2)(1) & (-2)(-1) & (-2)(2) \end{bmatrix}$

$= \begin{bmatrix} -2 & 2 & -4 \end{bmatrix}$

(viii) 3*D*

Solution: $3D$

As $D = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$

So, $3D$

$= (3) \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 3\times 1 & 3\times 2 & 3\times 3 \\ 3\times -1 & 3\times 0 & 3\times 2 \end{bmatrix}$

$$=\begin{bmatrix} 3 & 6 & 9 \\ -3 & 0 & 6 \end{bmatrix}$$

(ix) $3C$

Solution: $3C$

As $C=\begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$

So, 3C

$$=(3)\times\begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$$
$$=\begin{bmatrix} 3\times 1 & 3\times -1 & 3\times 2 \end{bmatrix}$$
$$=\begin{bmatrix} 3 & -3 & 6 \end{bmatrix}$$

Q.4 Perform the indicated operations and simplify the following:

(i) $\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}+\begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}\right)+\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

Solution: $\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}+\begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}\right)+\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

$$=\begin{bmatrix} 1+0 & 0+2 \\ 0+3 & 1+0 \end{bmatrix}+\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$
$$=\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}+\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$
$$=\begin{bmatrix} 1+1 & 2+1 \\ 3+1 & 1+0 \end{bmatrix}$$
$$=\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$$

(ii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}+\left(\begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}-\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\right)$

Solution:

$$=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}+\left(\begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}-\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\right)$$
$$=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}+\begin{bmatrix} 0-1 & 2-1 \\ 3-1 & 0-0 \end{bmatrix}$$
$$=\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}+\begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$$
$$=\begin{bmatrix} 1-1 & 0+1 \\ 0+2 & 1+0 \end{bmatrix}$$
$$=\begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$$

(iii) $\begin{bmatrix} 2 & 3 & 1 \end{bmatrix}+\left(\begin{bmatrix} 1 & 0 & 2 \end{bmatrix}-\begin{bmatrix} 2 & 2 & 2 \end{bmatrix}\right)$

Solution:

$$=\begin{bmatrix} 2 & 3 & 1 \end{bmatrix}+\begin{bmatrix} 1-2 & 0-2 & 2-2 \end{bmatrix}$$
$$=\begin{bmatrix} 2 & 3 & 1 \end{bmatrix}+\begin{bmatrix} -1 & -2 & 0 \end{bmatrix}$$
$$=\begin{bmatrix} 2-1 & 3-2 & 1-0 \end{bmatrix}$$
$$=\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

(iv) $\begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & -1 \\ 0 & 1 & 2 \end{bmatrix}+\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$

Solution:

$$=\begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & -1 \\ 0 & 1 & 2 \end{bmatrix}+\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$$
$$=\begin{bmatrix} 1+1 & 2+1 & 3+1 \\ -1+2 & -1+2 & -1+2 \\ 3+0 & 1+2 & 2-1 \end{bmatrix}$$
$$=\begin{bmatrix} 2 & 3 & 4 \\ 1 & 1 & 1 \\ 3 & 4 & 5 \end{bmatrix}$$

(v) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}+\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$

Solution:

$$=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}+\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$$
$$=\begin{bmatrix} 1+1 & 2+1 & 3+1 \\ -1+2 & -1+2 & -1+2 \\ 3+0 & 1+2 & 2-1 \end{bmatrix}$$
$$=\begin{bmatrix} 2 & 2 & 1 \\ 0 & 2 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

(vi) $\left(\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}+\begin{bmatrix}2 & 1\\1 & 0\end{bmatrix}\right)+\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$

Solution:

$$=\begin{bmatrix}1+2 & 2+1\\0+1 & 1+0\end{bmatrix}+\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}3 & 3\\1 & 1\end{bmatrix}+\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}3+1 & 3+1\\1+1 & 1+1\end{bmatrix}$$

$$=\begin{bmatrix}4 & 4\\2 & 2\end{bmatrix}$$

Q.5 For the matrices $A=\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$, $B=\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$ **and** $C=\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$, **verify the following rules:**

(i) $A+C=C+A$

Solutions:

L.H.S = A+C

R.H.S = C+A

LHS= A+C

$$=\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}1-1 & 2+0 & 3+0\\2+0 & 3-2 & 1+3\\1+1 & -1+1 & 0+2\end{bmatrix}$$

$$=\begin{bmatrix}0 & 2 & 3\\2 & 1 & 4\\2 & 0 & 2\end{bmatrix}$$

RHS= C+A

$$=\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}-1+1 & 0+2 & 0+3\\0+2 & -2+3 & 3+1\\1+1 & 1-1 & 2-0\end{bmatrix}$$

$$=\begin{bmatrix}0 & 2 & 3\\2 & 1 & 4\\2 & 0 & 2\end{bmatrix}$$

$A+C=C+A$

Hence proved

L.H.S = R.H.S

(ii) $A+B=B+A$

Solution: $A+B=B+A$

L.H.S = A+B

R.H.S = B+A

LHS= A+B

$$=\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1+1 & 2-1 & +3+1\\2+2 & 3-2 & 1+2\\1+3 & -1+1 & 0+3\end{bmatrix}$$

$$=\begin{bmatrix}2 & 1 & 4\\4 & 1 & 3\\4 & 0 & 3\end{bmatrix}$$

RHS= B+A

$$=\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\3 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1+1 & -1+2 & 1+3\\2+2 & -2+3 & 2+1\\3+3 & 1-1 & 3-0\end{bmatrix}$$

$$=\begin{bmatrix}2 & 1 & 4\\4 & 1 & 3\\6 & 0 & 3\end{bmatrix}$$

$A+B=B+A$

Hence proved

L.H.S = R.H.S

(iii) $B+C=C+B$

Solution: $B+C=C+B$

L.H.S = B+C

R.H.S = C+B

L.H.S = B+C

$$=\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}+\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix} 1-1 & -1+0 & 1+0 \\ 2+0 & -2-2 & 2+3 \\ 3+1 & 1+1 & 3+2 \end{bmatrix}$$

$$=\begin{bmatrix} 0 & -1 & 1 \\ 2 & -4 & 5 \\ 4 & 2 & 5 \end{bmatrix}$$

R.H.S = C+B

$$=\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}+\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$=\begin{bmatrix} -1+1 & 0-1 & 0+1 \\ 0+2 & -2-2 & 3+2 \\ 1+3 & 1+1 & 2+3 \end{bmatrix}$$

$$=\begin{bmatrix} 0 & -1 & 1 \\ 2 & -4 & 5 \\ 4 & 2 & 5 \end{bmatrix}$$

L.H.S = R.H.S
B+C=C+B
Hence proved

(iv) $A+(B+A)=2A+B$

Solution: $A+(B+A)=2A+B$

L.H.S = A+ (B+A)
R.H.S = 2A+B
L.H.S= A+ (B+A)

$$=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}+\left(\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}+\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}\right)$$

$$=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}+\begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix}$$

$$=\begin{bmatrix} 3 & 3 & 7 \\ 6 & 4 & 4 \\ 5 & -1 & 3 \end{bmatrix}$$

R.H.S = 2A+B

$$=2\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}+\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$=\begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \\ 2 & -2 & 0 \end{bmatrix}+\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$=\begin{bmatrix} 3 & 3 & 7 \\ 6 & 4 & 4 \\ 5 & -1 & 3 \end{bmatrix}$$

L.H.S =R.H.S
A+ (B+A) =2A+B
Hence proved

(v) $(C-B)+A=C+(A+B)$

Solution: $(C-B)+A=C+(A+B)$

L.H.S = (C-B) +A
R.H.S = C+(A-B)
L.H.S = (C-B) + A

$$=\left(\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}-\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}\right)+\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$=\begin{bmatrix} -2 & 1 & -1 \\ -2 & 0 & 1 \\ -2 & 0 & -1 \end{bmatrix}+\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$=\begin{bmatrix} -1 & 3 & 2 \\ 0 & 3 & 2 \\ -1 & -1 & -1 \end{bmatrix}$$

RHS= C+ (A-B)

$$=\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}+\left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}-\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}\right)$$

$$=\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}+\begin{bmatrix} 0 & 3 & 2 \\ 0 & 5 & -1 \\ -2 & -2 & -3 \end{bmatrix}$$

$$=\begin{bmatrix} -1 & 3 & 2 \\ 0 & 3 & 2 \\ -1 & -1 & -1 \end{bmatrix}$$

L.H.S = R.H.S
(C-B)+A=C+ (A-B)
Hence proved

(vi) $2A+B=A+(A+B)$

Solution: $2A+B=A+(A+B)$

L.H.S = 2A+B
R.H.S = A+ (A+B)
LHS= 2A+B

$$=2\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}2 & 4 & 6\\4 & 6 & 2\\2 & -2 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}3 & 3 & 7\\6 & 4 & 4\\5 & -1 & 3\end{bmatrix}$$

RHS= A + (A+B)

$$=\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\left(\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}\right)$$

$$=\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\begin{bmatrix}2 & 1 & 4\\4 & 1 & 3\\4 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}3 & 3 & 7\\6 & 4 & 4\\5 & -1 & 3\end{bmatrix}$$

L.H.S = R.H.S

2A+B=A+ (A+B)

Hence proved

(vii) $(C-B)-A=(C-A)-B$

Solution: $(C-B)-A=(C-A)-B$

L.H.S = (C-B) –A

R.H.S = (C-A) –B

LHS= (C-B) – A

$$=\left(\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}-\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}\right)-\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & - & 0\end{bmatrix}$$

$$=\begin{bmatrix}-2 & 1 & -1\\-2 & 0 & 1\\-2 & 0 & -1\end{bmatrix}-\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}-3 & -1 & -4\\-4 & -3 & 0\\-3 & 1 & -1\end{bmatrix}$$

RHS= (C-A) –B

$$=\left(\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}-\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}\right)-\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-2 & -2 & -3\\-2 & -5 & 2\\0 & 2 & 2\end{bmatrix}-\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-3 & -1 & -4\\-4 & -3 & 0\\-3 & 1 & -1\end{bmatrix}$$

L.H.S = R.H.S

(C-B)-A=(C-A)-B

Hence proved

(viii) $(A+B)+C=A+(B+C)$

Solution: (A+B) + C = A+ (B+C)

L.H.S = (A+B) + C

R.H.S = A + (B+C)

LHS= (A+B) +C

$$=\left(\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}\right)+\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}2 & 1 & 4\\4 & 1 & 3\\4 & 0 & 3\end{bmatrix}+\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}1 & 1 & 4\\4 & -1 & 6\\5 & 1 & 5\end{bmatrix}$$

R.H.S = A+ (B+C)

$$=\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\3 & -1 & 0\end{bmatrix}+\left(\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}+\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}\right)$$

$$=\begin{bmatrix}2 & 1 & 4\\4 & 1 & 3\\4 & 0 & 3\end{bmatrix}+\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}1 & 1 & 4\\4 & -1 & 6\\5 & 1 & 5\end{bmatrix}$$

L.H.S = R.H.S
(A+B)+C=A+ (B+C)
Hence proved

(ix) $A+(B-C)=(A-C)+B$

Solution: $A+(B-C)=(A-C)+B$

L.H.S = A+ (B-C)
R.H.S = (A-C) + B
L.H.S = A+ (B-C)

$$=\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\left(\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}-\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}\right)$$

$$=\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\begin{bmatrix}2 & -1 & 1\\2 & 0 & -1\\2 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}3 & 1 & 4\\4 & 3 & 0\\3 & -1 & 1\end{bmatrix}$$

RHS= (A-C) +B

$$=\left(\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}-\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}\right)+\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1+1 & 2-0 & 3-1\\2-0 & 3+2 & 1-3\\1-1 & -1-1 & 0-2\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}2 & 2 & 3\\2 & 5 & -2\\0 & -2 & -2\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}3 & 1 & 4\\4 & 3 & 0\\3 & -1 & 1\end{bmatrix}$$

L.H.S = R.H.S
A+ (B-C) = (A-C) +B
Hence proved

(x) $2A+2B=2(A+B)$

Solution: $2A+2B=2(A+B)$

L.H.S = 2A + 2B
R.H.S = 2(A + B)
L.H.S = 2A + 2B

$$=2\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+2\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}2 & 4 & 6\\4 & 6 & 2\\2 & -2 & 0\end{bmatrix}+\begin{bmatrix}2 & -2 & 2\\4 & -4 & 4\\6 & 2 & 6\end{bmatrix}$$

$$=\begin{bmatrix}4 & 2 & 8\\8 & 2 & 6\\8 & 0 & 6\end{bmatrix}$$

RHS= 2 (A+B)

$$=2\left(\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}\right)$$

$$=\begin{bmatrix}1+1 & 2-1 & 3+1\\2+2 & 3-2 & 1+2\\1+3 & -1+1 & 0+3\end{bmatrix}$$

$$=2\begin{bmatrix}2 & 1 & 4\\4 & 1 & 3\\4 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}4 & 2 & 8\\8 & 2 & 6\\8 & 0 & 6\end{bmatrix}$$

L.H.S = R.H.S
2A+2B=2(A+B)
Hence proved

Q.6 **If** $A=\begin{bmatrix}1 & -2\\3 & 4\end{bmatrix}$ **and** $B=\begin{bmatrix}0 & 7\\-3 & 8\end{bmatrix}$

find:

(i) $3A-2B$

Solution: $3A-2B$

$$3A-2B=3\begin{bmatrix}1 & -2\\3 & 4\end{bmatrix}-2\begin{bmatrix}0 & 7\\-3 & 8\end{bmatrix}$$

$$=\begin{bmatrix}3 & -6\\9 & 12\end{bmatrix}-\begin{bmatrix}0 & 14\\-6 & 16\end{bmatrix}$$

$$=\begin{bmatrix}3 & -20\\15 & -4\end{bmatrix}$$

(ii) $2A^t-3B^t$

Solution: $2A^t-3B^t$

When we take transpose of any matrix we change rows into columns or columns into rows.

$$A^t=\begin{bmatrix}1 & 3\\-2 & 4\end{bmatrix}$$

$$B^t=\begin{bmatrix}0 & -3\\7 & 8\end{bmatrix}$$

$$2A^t-3B^t=2\begin{bmatrix}1 & 3\\-2 & 4\end{bmatrix}-3\begin{bmatrix}0 & -3\\7 & 8\end{bmatrix}$$

$$=\begin{bmatrix}2 & 6\\-4 & 8\end{bmatrix}-\begin{bmatrix}0 & -9\\21 & 24\end{bmatrix}$$

$$=\begin{bmatrix}2 & 15\\-25 & -16\end{bmatrix}$$

Q.7 If

$$2\begin{bmatrix}2 & 4\\-3 & a\end{bmatrix}+3\begin{bmatrix}1 & b\\8 & -4\end{bmatrix}=\begin{bmatrix}7 & 10\\18 & 1\end{bmatrix}$$

Solution:

$$2\begin{bmatrix}2 & 4\\-3 & a\end{bmatrix}+3\begin{bmatrix}1 & b\\8 & -4\end{bmatrix}=\begin{bmatrix}7 & 10\\18 & 1\end{bmatrix}$$

$$=\begin{bmatrix}4 & 8\\-6 & 2a\end{bmatrix}+\begin{bmatrix}3 & 3b\\24 & -12\end{bmatrix}=\begin{bmatrix}7 & 10\\18 & 1\end{bmatrix}$$

$$=\begin{bmatrix}7 & 8+3b\\18 & 2a+(-12)\end{bmatrix}=\begin{bmatrix}7 & 10\\18 & 1\end{bmatrix}$$

$8 + 3b = 10$ __________ (i)

$2a - 12 = 1$ ___________ (ii)

By solving equation (ii) we get the value of a

$2a - 12 = 1$

$2a = 1 + 12$

$2a = 13$

$a=\dfrac{13}{2}$

By solving equation (i) we get the value of *b*

$8 + 3b = 10$

$3b = 10 - 8$

$3b = 2$

$b=\dfrac{2}{3}$

Q.8 **If** $A=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}$ **and** $B=\begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}$

Then verify that

(i) $(A+B)^t=A^t+B^t$

Solution: $(A+B)^t=A^t+B^t$

L.H.S = $(A + B)^t$

R.H.S = $A^t + B^t$

To solve L.H.S

L.H.S = $(A + B)^t$

$$=(A+B)^t=\left(\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}+\begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}\right)^t$$

$$=\begin{bmatrix}2 & 3\\2 & 1\end{bmatrix}^t$$

$$\text{R.H.H}=(A+B)^t=\begin{bmatrix}2 & 2\\3 & 1\end{bmatrix}$$

To solve R.H.S

R.H.S = $A^t + B^t$

$$A^t=\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$$

$$B^t=\begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$$

$$\text{RHS}=A^t+B^t=\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}+\begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}2 & 2\\3 & 1\end{bmatrix}$$

L.H.S = R.H.S $\Rightarrow (A+B)^t=A^t+B^t$

Hence Proved

(ii) $(A-B)^t=A^t-B^t$

Solution: $(A-B)^t=A^t-B^t$

L.H.S = $(A - B)^t$

R.H.S = $A^t - B^t$

LHS= $(A - B)^t$

$$(A - B)^t = \left(\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}\right)^t$$

$$= \begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix}^t$$

$$(A - B)^t = \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$$

R.H.S = $A^t - B^t$

$$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^t - \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 0-2 \\ 2-1 & 1-0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$$

L.H.S = R.H.S

$(A-B)^t=A^t-B^t$

Hence proved

(iii) $A + A^+$ **is a symmetric**

Solution:

$A + A^+$ is a symmetric

To show that $A+A^t$ is symmetric, we will show that

$$\left(A + A^t\right)^t = \left(A + A^t\right)$$

$$A + A^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 2+0 \\ 0+2 & 1+1 \end{bmatrix}$$

$$A+A^t = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$(A+A^t)^t = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}^t$$

$$= \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$(A+A^t)^t=(A+A^t)$

Hence Proved

$A+A^t$ symmetric

(iv) $A - A^t$ **is a skew symmetric**

Solution: $A - A^t$

To show that A-A^t is skew symmetric we will show that

$(A-A^t)^t=-(A-A^t)$

$$A - A^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^t - \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 2-0 \\ 0-2 & 1-1 \end{bmatrix}$$

$$A-A^t = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$(A-A^t)^t = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

$$=- \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$(A-A^t)^t=-(A-A^t)$

Hence proved

A – A^t is a skew symmetric

(v) $B + B^t$ **is a symmetric**

Solution: $B + B^t$

The show that $B+B^t$ is symmetric we will show that

$$(B + B^t)^t = (B + B^t)$$

$$B + B^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 1+2 \\ 2+1 & 0+0 \end{bmatrix}$$

$$B + B^t = \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix}$$

$$\left(B + B^t\right)^t = \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix}$$

$$\left(B + B^t\right)^t = \left(B + B^t\right)$$

Hence proved
$B+B^t$ is a symmetric

(vi) $B-B^t$ **is a skew symmetric**
Solution: $B-B^t$
To show that $B - B^t$ is skew symmetric, we will show that
$(B-B^t)^t = -(B-B^t)$

$$B-B^t=\begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}-\begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}^t$$
$$=\begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}-\begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$$
$$=\begin{bmatrix}1-1 & 1-2\\2-1 & 0-0\end{bmatrix}$$
$$B-B^t=\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$$
$$(B-B^t)^t=\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}^t$$
$$=\begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}$$
$$-\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$$
$$(B-B^t)^t=-(B-B^t)$$

Hence proved
$B-B^t$ is a skew symmetric.

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Report any mistake at freeilm786@gmail.com

Exercise 1.4

Q.1 Which of the following product of matrices if conformable for multiplication?

(i) $\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix}$

Yes, these matrices can be multiplied because number of columns of 1^{st} matrix is equal to number of rows of 2^{nd} matrix.

(ii) $\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} -2 & -1 \\ 1 & 3 \end{bmatrix}$

Yes, these matrices can be multiplied because number of columns of 1^{st} matrix is equal to number of rows of 2^{nd} matrix.

(iii) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$

No, these matrices cannot be multiplied because number of columns of 1^{st} matrix is not equal to the number of rows of 2^{nd} matrix.

(iv) $\begin{bmatrix} 1 & 2 \\ 0 & -1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$

Yes, these matrices can be multiplied because number of columns of 1^{st} matrix is equal to number of rows of 2^{nd} matrix.

(v) $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ -2 & 3 \end{bmatrix}$

Yes, these matrices can be multiplied because number of columns of 1^{st} matrix is equal to number of rows of 2^{nd} matrix.

Q.2 If $A=\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$, $B=\begin{bmatrix} 6 \\ 5 \end{bmatrix}$ **find**

(i) AB

Solution: $AB=\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 6 \\ 5 \end{bmatrix}$

$$AB=\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

$$=\begin{bmatrix} (3\times 6)+(0\times 5) \\ (-1\times 6)+(2\times 5) \end{bmatrix}$$

$$=\begin{bmatrix} 18+0 \\ -6+10 \end{bmatrix}=\begin{bmatrix} 18 \\ 4 \end{bmatrix}$$

(ii) BA **(if possible)**

Solution:

BA is not possible became number of columns of B not equal to number of rows of A.

Q.3 Find the following products

(i) $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$

Solution: $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$

$= \begin{bmatrix}(1\times4)+(2\times0)\end{bmatrix}$

$= \begin{bmatrix}4+0\end{bmatrix}$

$= \begin{bmatrix}4\end{bmatrix}$

(ii) $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}5\\-4\end{bmatrix}$

Solution: $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}4\\-0\end{bmatrix}$

$= \begin{bmatrix}(1\times5)+(2\times-4)\end{bmatrix}$

$= \begin{bmatrix}5+(-8)\end{bmatrix}$

$= [5-8]$

$= [-3]$

(iii) $\begin{bmatrix}-3 & 0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$

Solution: $\begin{bmatrix}-3 & 0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$

$= \begin{bmatrix}(-3\times4)+(0\times0)\end{bmatrix}$

$= [-12+0]$

$= [-12]$

(iv) $\begin{bmatrix}6 & 0\end{bmatrix}\begin{bmatrix}4\\-0\end{bmatrix}$

Solution: $\begin{bmatrix}6 & 0\end{bmatrix}\begin{bmatrix}4\\-0\end{bmatrix}$

$\begin{bmatrix}6 & +0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$

$= \begin{bmatrix}6\times4+(-0)(0)\end{bmatrix}$

$= \begin{bmatrix}24-0\end{bmatrix}$

$= \begin{bmatrix}24\end{bmatrix}$

(v) $\begin{bmatrix}1 & 2\\-3 & 0\\6 & -1\end{bmatrix}\begin{bmatrix}4 & 5\\0 & -4\end{bmatrix}$

Solution: $\begin{bmatrix}1 & 2\\-3 & 0\\6 & -1\end{bmatrix}\begin{bmatrix}4 & 5\\0 & -4\end{bmatrix}$

$= \begin{bmatrix}1\times4+2\times0 & 1\times5+2\times(-4)\\-3\times4+0\times0 & -3(5)+0\times(-4)\\6(4)+(-1)(0) & 6(5)+(-1)(-4)\end{bmatrix}$

$= \begin{bmatrix}4+0 & 5-8\\-12+0 & -15-0\\24-0 & 30+4\end{bmatrix}$

$= \begin{bmatrix}4 & -3\\-12 & -15\\24 & 34\end{bmatrix}$

Q.4 Multiply the following matrices.

(a) $\begin{bmatrix}2 & 3\\1 & 1\\0 & -2\end{bmatrix}\begin{bmatrix}2 & -1\\3 & 0\end{bmatrix}$

Solution: $\begin{bmatrix}2 & 3\\1 & 1\\0 & -2\end{bmatrix}\begin{bmatrix}2 & -1\\3 & 0\end{bmatrix}$

$\begin{bmatrix}2\times2+(3\times3) & (2\times-1)+(3\times0)\\(1\times2)+(1\times3) & (1\times-1)+(1\times0)\\(0\times2)+(-2\times3) & (0\times-1)+(-2\times0)\end{bmatrix}$

$= \begin{bmatrix}4+9 & -2+0\\2+3 & -1+0\\0+-6 & 0+0\end{bmatrix}$

$$=\begin{bmatrix} 13 & -2 \\ 5 & -1 \\ 0-6 & 0 \end{bmatrix}$$

$$=\begin{bmatrix} 13 & -2 \\ 5 & -1 \\ -6 & 0 \end{bmatrix}$$

(b) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix}$

Solution: $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix}$

$$=\begin{bmatrix} (1\times1)+(2\times3)+(3\times-1) & (1\times2)+(2\times4)+(3\times1) \\ (4\times1)+(5\times3)+(6\times-1) & (4\times2)+(5\times4)+(6\times1) \end{bmatrix}$$

$$=\begin{bmatrix} 1+6+(-3) & 2+8+3 \\ 4+15+(-6) & 8+20+6 \end{bmatrix}$$

$$=\begin{bmatrix} 7-3 & 13 \\ 19-6 & 34 \end{bmatrix}$$

$$=\begin{bmatrix} 4 & 13 \\ 13 & 34 \end{bmatrix}$$

(c) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

Solution: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

$$=\begin{bmatrix} (1\times1)+(2\times4) & (1\times2)+(2\times5) & (1\times3)+(2\times6) \\ (3\times1)+(4\times4) & (3\times2)+(4\times5) & (3\times3)+(4\times6) \\ (-1\times1)+(1\times4) & (-1\times2)+(1\times5) & (-1\times3)+(1\times6) \end{bmatrix}$$

$$=\begin{bmatrix} 1+8 & 2+10 & 3+12 \\ 3+16 & 6+20 & 9+24 \\ -1+4 & -2+5 & -3+6 \end{bmatrix}$$

$$=\begin{bmatrix} 9 & 12 & 15 \\ 19 & 26 & 33 \\ 3 & 3 & 3 \end{bmatrix}$$

(d) $\begin{bmatrix} 8 & 5 \\ 6 & 4 \end{bmatrix}\begin{bmatrix} 2 & -\frac{5}{2} \\ -4 & 4 \end{bmatrix}$

Solution: $\begin{bmatrix} 8 & 5 \\ 6 & 4 \end{bmatrix}\begin{bmatrix} 2 & -\frac{5}{2} \\ -4 & 4 \end{bmatrix}$

$$=\begin{bmatrix} (8\times2)+(5\times-4) & \left(8\times-\frac{5}{2}\right)+(5\times4) \\ (6\times2)+(4\times-4) & \left(6\times-\frac{5}{2}\right)+(4\times4) \end{bmatrix}$$

$$=\begin{bmatrix} 16+(-20) & \frac{-40}{2}+20 \\ 12+(-16) & \frac{-30}{2}+16 \end{bmatrix}$$

$$=\begin{bmatrix} 16-20 & -20+20 \\ 12-16 & -15+16 \end{bmatrix}$$

$$=\begin{bmatrix} -4 & 0 \\ -4 & 1 \end{bmatrix}$$

(e) $\begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Solution: $\begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$=\begin{bmatrix} (-1\times0)+(2\times0) & (-1\times0)+(2\times0) \\ (1\times0)+(3\times0) & (1\times0)+(3\times0) \end{bmatrix}$$

$$=\begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Q.5 **Let** $A=\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$, $B=\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$ **and** $C=\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ **verify whether**

(i) $AB = BA$

Solution: $AB = BA$

L.H.S = AB

R.H.S = BA

L.H.S =AB

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+(3\times-5)\\ (2\times1)+(0-3) & (2\times2)+(0-5)\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-9) & -2+(-15)\\ 2+0 & 4+0\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2 & 4\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}$$

$$\text{R.H.S} = \text{BA} = \begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}\times\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1\times(-1)+2\times2 & 1\times3+2\times0\\ -3\times(-1)+(-5)2 & -3\times3+(-5)(0)\end{bmatrix}$$

$$=\begin{bmatrix}-1+4 & 3+0\\ 3-10 & -9-0\end{bmatrix}$$

$$=\begin{bmatrix}3 & 3\\ -7 & -9\end{bmatrix}$$

Since L.H.S $\neq$ R.H.S

L.H.S $\neq$ R.H.S

L.H.S $\neq$ R.H.S

(ii) $A(BC)=(AB)C$

Solution: $A(BC)=(AB)C$

L.H.S = A (BC)

R.H.S = (AB) C

L.H.S

L.H.S=A(BC)

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\left(\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}\right)$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}2+2 & 1+6\\ -6+(-5) & -3+(-15)\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}4 & 7\\ -6-5 & -3-15\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}4 & 7\\ -11 & -18\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times4)+(3\times-11) & (-1\times7)+(3\times-18)\\ (2\times4)+(0\times-11) & (2\times7)+(0\times-18)\end{bmatrix}$$

$$=\begin{bmatrix}-4+(-33) & -7+(-54)\\ 8+0 & 14+0\end{bmatrix}$$

$$=\begin{bmatrix}-4-33 & -7-54\\ 8 & 14\end{bmatrix}$$

$$=\begin{bmatrix}-37 & -61\\ 8 & 14\end{bmatrix}$$

R.H.S = (AB)C

$$=\left(\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}\right)\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+(-3\times-5)\\ (2\times1)+(0\times-3) & (2\times2)+(0\times-5)\end{bmatrix}$$

$$=\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-9) & -2+(-15)\\ 2+0 & -4+0\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2 & 4\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}(-10\times2)+(-17\times1) & (-10\times1)+(-17\times3)\\ (2\times2)+(4\times1) & (2\times1)(4\times3)\end{bmatrix}$$

$$=\begin{bmatrix}-20+(-17) & -10+(-51)\\ 4+4 & 2+12\end{bmatrix}$$

$$=\begin{bmatrix}-20-17 & -10-51\\ 8 & 14\end{bmatrix}$$

$$=\begin{bmatrix}-37 & -61\\ 8 & 14\end{bmatrix}$$

Since

L.H.S = R.H.S $\Rightarrow$ A (BC) = (AB)C

Hence proved

(iii) $A(B+C)=AB+AC$

Solution: $A(B+C)=AB+AC$

L.H.S = A (B+C)

R.H.S = AB+AC

L.H.S

LHS=A (B+C)

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\left(\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}+\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}\right)$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1+2 & 2+1\\ -3+1 & -5+3\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}3 & 3\\ -2 & -2\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times3)+(3\times-2) & (-1\times3)+(3\times-2)\\ (2\times3)+(0\times-2) & (2\times3)+(0\times-2)\end{bmatrix}$$

$$=\begin{bmatrix}-3+(-6) & -3+(-6)\\ 6+0 & 6+0\end{bmatrix}$$

$$=\begin{bmatrix}-3-6 & -3-6\\ 6 & 6\end{bmatrix}$$

$$=\begin{bmatrix}-9 & -9\\ 6 & 6\end{bmatrix}$$

R.H.S=AB+AC

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}+\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+(3\times-5)\\ (2\times1)+(0\times-3) & (2\times+2)+(0\times-5)\end{bmatrix}$$

$$+\begin{bmatrix}(-1\times2)+(3\times1) & (-1\times1)+(3\times3)\\ (2\times2)+(0\times1) & (2\times1)+(0\times3)\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-3) & -2+(-15)\\ 2+0 & +4+0\end{bmatrix}+\begin{bmatrix}-2+3 & -1+9\\ 4+0 & 2+0\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2 & +4\end{bmatrix}+\begin{bmatrix}1 & 8\\ 4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & +4\end{bmatrix}+\begin{bmatrix}1 & 8\\ 4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-10+1 & -17+8\\ 2+4 & +4+2\end{bmatrix}$$

$$=\begin{bmatrix}-9 & -9\\ 6 & 6\end{bmatrix}$$

Since LHS=RHS

A (B+C) = AB+AC

Hence proved

(iv) A(B–C) = AB–AC

Solution: A (B–C) = AB–AC

L.H.S = A (B–C)

R.H.S = AB–AC

L.H.S=A(B–C)

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\left(\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}-\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}\right)$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1-2 & 2-1\\ -3-1 & -5-3\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}-1 & 1\\ -4 & -8\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times-1)+(3\times-4) & (-1\times1)+(3\times-8)\\ (2\times-1)+(0l-4) & (2\times1)+(0\times-8)\end{bmatrix}$$

$$=\begin{bmatrix}+1+(-12) & -1+(-24)\\ -2+0 & 2+0\end{bmatrix}$$

$$=\begin{bmatrix}1-12 & -1-24\\ -2 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-11 & -25\\ -2 & 2\end{bmatrix}$$

R.H.S=AB–AC

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}-\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+3\times-5)\\ (2\times1)+(0\times-3) & (2\times2)+(0\times-5)\end{bmatrix}$$

$$-\begin{bmatrix}(-1\times2)+(3\times1) & (-1\times1)+(3\times3)\\ (2\times2)+(0\times1) & (2\times1)+(0\times3)\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)-(3\times3) & (-1\times2)+(3\times-5)\\ (2\times1)+(0\times-3) & (2\times2)+(0\times-5)\end{bmatrix}$$

$$-\begin{bmatrix}(-1\times2)+(3\times1) & (-1\times1)+(3\times3)\\ (2\times2)+(0\times1) & (2\times1)+(0\times3)\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2+0 & 4+0\end{bmatrix}-\begin{bmatrix}-2+3 & -1+9\\ 4+0 & 2+0\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}-\begin{bmatrix}1 & 8\\ 4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-10-1 & -17-8\\ 2-4 & 4-2\end{bmatrix}$$

$$=\begin{bmatrix}-11 & -25\\ -2 & 2\end{bmatrix}$$

Since L.H.S = R.H.S

A (B-C) =AB-AC, Hence **proved.**

Q.6 For the matrices $A=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$, $B=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$ **and** $C=\begin{bmatrix}-2 & 6\\ 3 & -9\end{bmatrix}$

Verify that

(i) $(AB)^t = B^t A^t$

Solution: $(AB)^t = B^t A^t$

L.H.S $= (AB)^t$

R.H.S $= B^t A^t$

$$(AB)=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+(3\times-5)\\ (+2\times1)+(0\times-3) & (2\times2)+(0\times-5)\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-9) & -2+(-15)\\ 2+0 & 4+0\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2 & 4\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}$$

LHS= $(AB)^t$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}^t$$

$$=\begin{bmatrix}-10 & 2\\ -17 & 4\end{bmatrix}$$

$$B^t=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}^t$$

$$B^t=\begin{bmatrix}1 & -3\\ 2 & -5\end{bmatrix}$$

$$A^t=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$$

$$A^t=\begin{bmatrix}-1 & 2\\ 3 & 0\end{bmatrix}$$

L.H.S=B^tA^t

$$=\begin{bmatrix}1 & -3\\ 2 & -5\end{bmatrix}\times\begin{bmatrix}-1 & 2\\ 3 & 0\end{bmatrix}$$

$$=\begin{bmatrix}(1\times-1)+(-3\times3) & (1\times2)+(-3\times0)\\ (2\times-1)+(-5\times3) & (2\times2)+(-5\times0)\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-9) & 2+0\\ -2+(-15) & 4+0\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & 2\\ -2-15 & 4\end{bmatrix}$$

$$=\begin{bmatrix}-10 & 2\\ -17 & 4\end{bmatrix}$$

Since L.H.S = R.H.S

$(AB)^t = B^tA^t$

Hence proved

L.H.S = R.H.S

(ii) $\mathbf{(BC)^t= C^tB^t}$

Solution: $(BC)^t= C^tB^t$

L.H.S $= (BC)^t$

R.H.S $= C^t B^t$

To solve L.H.S

$$BC=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}\times\begin{bmatrix}-2 & 6\\ 3 & -9\end{bmatrix}$$

$$=\begin{bmatrix}(1\times-2)+(2\times3) & (1\times6)+(2\,x\times-9)\\ (-3\times-2)+(-5\times3) & (-3\times6)+(-5\times-9)\end{bmatrix}$$

$$=\begin{bmatrix}-2+6 & 6+(-18)\\ 6+(-15) & -18+45\end{bmatrix}$$

$$=\begin{bmatrix}4 & 6-18\\ 6-15 & 27\end{bmatrix}$$

$$=\begin{bmatrix}4 & -12\\ -9 & 27\end{bmatrix}$$

Taking transpose of BC:-

$$(BC)^t=\begin{bmatrix}4 & -12\\ -9 & 27\end{bmatrix}^t$$

$$LHS = (B\ C)^t =\begin{bmatrix}4 & -9\\ -12 & 27\end{bmatrix}$$

To solve R.H.S =

Taking transpose of matrix C

$$C^t=\begin{bmatrix}-2 & 3\\ 6 & -9\end{bmatrix}$$

Taking transpose of matrix B

$$B^t=\begin{bmatrix}1 & -3\\ 2 & -5\end{bmatrix}$$

Now, multiplying matrices, $B^t C^t$

$$R.H.S = C^t B^t=\begin{bmatrix}-2 & 3\\ 6 & -9\end{bmatrix}\times\begin{bmatrix}1 & -3\\ 2 & -5\end{bmatrix}$$

$$=\begin{bmatrix}(-2\times1)+(3\times2) & (-2\times-3)+(3\times-5)\\ (6\times1)+(-9\times2) & (6\times-3)+(-9\times-5)\end{bmatrix}$$

$$=\begin{bmatrix} -2+6 & 6+(-15) \\ 6+(-18) & -18+45 \end{bmatrix}$$

$$=\begin{bmatrix} 4 & 6-15 \\ 6-18 & 27 \end{bmatrix}$$

$$=\begin{bmatrix} 4 & -9 \\ -12 & 27 \end{bmatrix}$$

Hence proved

L.H.S = R.H.S

Last Updated: September 2020

Report any mistake at freeilm786@gmail.com

Exercise 1.5

Q.1 Find the determinant of following matrices.

(i) $A=\begin{bmatrix}-1 & 1\\ 2 & 0\end{bmatrix}$

Solution:

$A=\begin{bmatrix}-1 & 1\\ 2 & 0\end{bmatrix}$

To write the determinant form

$|A|=\begin{vmatrix}-1 & 1\\ 2 & 0\end{vmatrix}$

$= (-1)(0) - (2)(1)$
$= 0 - 2$
$= -2$

(ii) $B=\begin{bmatrix}1 & 3\\ 2 & -2\end{bmatrix}$

Solution:

$B=\begin{bmatrix}1 & 3\\ 2 & -2\end{bmatrix}$

To write in determinant form

$|B|=\begin{vmatrix}1 & 3\\ 2 & -2\end{vmatrix}$

$= (1)(-2) - (2)(3)$
$= -2-6$
$= -8$

(iii) $C=\begin{bmatrix}3 & 2\\ 3 & 2\end{bmatrix}$

Solution:

$C=\begin{bmatrix}3 & 2\\ 3 & 2\end{bmatrix}$

To write in determinant form

$|C|=\begin{vmatrix}3 & 2\\ 3 & 2\end{vmatrix}$

$= (3)(2) - (3)(2)$
$= 6-6$
$= 0$

(iv) $D=\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}$

Solution:

$D=\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}$

To write in determinant form

$|D|=\begin{vmatrix}3 & 2\\ 1 & 4\end{vmatrix}$

$= (3)(4) - (2)(1)$
$= 12-2$
$= 10$

Q.2 Find which of the following matrices are singular or non-singular?

(i) $A=\begin{bmatrix}3 & 6\\ 2 & 4\end{bmatrix}$

Solution:

$A=\begin{bmatrix}3 & 6\\ 2 & 4\end{bmatrix}$

To write in determinant form

$|A|=\begin{vmatrix}3 & 6\\ 2 & 4\end{vmatrix}$

$|A| = (3)(4)-(2)(6)$

$|A| = 12-12$

$|A| = 0$

It is a singular matrix.

(ii) $B=\begin{bmatrix}4 & 1\\ 3 & 2\end{bmatrix}$

Solution:

$B=\begin{bmatrix}4 & 1\\ 3 & 2\end{bmatrix}$

To write in determinant form

$|B| = \begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix}$

$|B| = (4)\ (2) - (3)(1)$

$|B| = 8 - 3$

$|B| = 5$

It is non-singular matrix.

(iii) $C = \begin{bmatrix} 7 & -9 \\ 3 & 5 \end{bmatrix}$

Solution:

$C = \begin{bmatrix} 7 & -9 \\ 3 & 5 \end{bmatrix}$

To write in determinant form

$|C| = \begin{vmatrix} 7 & -9 \\ 3 & 5 \end{vmatrix}$

$|C| = (7)(5) - (3)(-9)$

$|C| = 35 + 27$

$|C| = 62$

In not equal to zero so
It is non-singular matrix.

(iv) $D = \begin{bmatrix} 5 & -10 \\ -2 & 4 \end{bmatrix}$

Solution:

$D = \begin{bmatrix} 5 & -10 \\ -2 & 4 \end{bmatrix}$

To write in determinant form

$|D| = \begin{vmatrix} 5 & -10 \\ -2 & 4 \end{vmatrix}$

$|D| = (5)(4) - (-2)(-10)$

$|D| = 20\text{-}20$

$|D| = 0$

It is singular matrix.

Q.3 Find the multiplicative inverse of each

(i) $A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$

Solution:

$A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$

To write in determinant form

$|A| = \begin{vmatrix} -1 & 3 \\ 2 & 0 \end{vmatrix}$

$|A| = (-1)(0) - (2)(3)$

$|A| = 0 - 6$

$|A| = -6 \neq 0$ (Non-Singular)

A^{-1}exists

To write in Adj A

$AdjA = \begin{bmatrix} 0 & -3 \\ -2 & -1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \times AdjA$

Putting the values

$$A^{-1} = \frac{1}{-6} \times \begin{bmatrix} 0 & -3 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 0 \times \frac{1}{-6} & -3 \times \frac{1}{-6} \\ -2 \times \frac{1}{-6} & -1 \times \frac{1}{-6} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{0}{-6} & \frac{+3}{+6} \\ \frac{+2}{+6} & \frac{+1}{+6} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{6} \end{bmatrix}$$

(ii) $B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$

Solution:

$B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$

To write in determinant form

$|B|=\begin{vmatrix}1 & 2\\ -3 & -5\end{vmatrix}$

$|B|=(-1)(-5)-(-3)(2)$

$|B|=-5+6$

$|B|=1\neq 0$ (Non-Singular)

B^{-1} exists

$AdjB=\begin{bmatrix}-5 & -2\\ 3 & 1\end{bmatrix}$

$B^{-1}=\dfrac{1}{|B|}\times AdjB$

Putting the values

$B^{-1}=\dfrac{1}{1}\times\begin{bmatrix}-5 & -2\\ 3 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{2}\times -5 & \frac{1}{1}\times -2\\ \frac{1}{1}\times 3 & \frac{1}{1}\times 1\end{bmatrix}$

$=\begin{bmatrix}\frac{-5}{1} & \frac{-2}{1}\\ \frac{3}{1} & \frac{1}{1}\end{bmatrix}$

$=\begin{bmatrix}-5 & -2\\ 3 & 1\end{bmatrix}$

(iii) $C=\begin{bmatrix}-2 & 6\\ 3 & -9\end{bmatrix}$

Solution:

To write in determinant form

$|C|=\begin{vmatrix}-2 & 6\\ 3 & -9\end{vmatrix}$

$|C|=(-2)(-9)-(3)(6)$

$|C|=18-18$

$|C|=0$ Singular

C^{-1} Does not exists.

(iv) $D=\begin{bmatrix}\frac{1}{2} & \frac{3}{4}\\ 1 & 2\end{bmatrix}$

Solution:

To write in determinant form

$D=\begin{bmatrix}\frac{1}{2} & \frac{3}{4}\\ 1 & 2\end{bmatrix}$

$|D|=\begin{vmatrix}\frac{1}{2} & \frac{3}{4}\\ 1 & 2\end{vmatrix}=\frac{1}{2}\times 2-\frac{3}{4}\times 1$

$=1-\frac{3}{4}$

$=\dfrac{4-3}{4}$

$|D|=\frac{1}{4}\neq 0$ (Non Singular)

D^{-1} exists

$AdjD=\begin{bmatrix}2 & \frac{-3}{4}\\ -1 & \frac{1}{2}\end{bmatrix}$

$D^{-1}=\dfrac{1}{|D|}\times AdjD$

By putting the values

$=\dfrac{1}{\frac{1}{4}}\begin{bmatrix}2 & -\frac{3}{4}\\ -1 & \frac{1}{2}\end{bmatrix}$

$=1\div\frac{1}{4}\begin{bmatrix}2 & \frac{-3}{4}\\ -1 & \frac{1}{2}\end{bmatrix}$

$=1\times\frac{4}{1}\begin{bmatrix}2 & \frac{-3}{2}\\ -1 & \frac{1}{2}\end{bmatrix}$

$=4\begin{bmatrix}2 & -\frac{3}{4}\\ -1 & \frac{1}{2}\end{bmatrix}$

$=\begin{bmatrix}8 & -3\\ -4 & 2\end{bmatrix}$

Q.4 **If** $A=\begin{bmatrix}1 & 2\\ 4 & 6\end{bmatrix}$ **and** $B=\begin{bmatrix}3 & -1\\ 2 & -2\end{bmatrix}$, **then**

Then verify that

(i) A(AdjA)=(AdjA)A=(detA)I

Solution: A(AdjA)=(AdjA)A=(detA)I

$$A=\begin{bmatrix}1 & 2\\4 & 6\end{bmatrix}$$

$$\text{AdjA}=\begin{bmatrix}6 & -2\\-4 & 1\end{bmatrix}$$

$$\text{detA}=\begin{bmatrix}1 & 2\\4 & 6\end{bmatrix}$$

$=1\times 6 \quad 2\times 4$

$=6-8$

$=-2$

$$A(AdjA)=\begin{bmatrix}1 & 2\\4 & 6\end{bmatrix}\begin{bmatrix}6 & -2\\-4 & 1\end{bmatrix}$$

$$=\begin{bmatrix}6-8 & (-2)+2\\24-4 & -8+6\end{bmatrix}$$

$$\text{A (Adj A)}=\begin{bmatrix}-2 & 0\\0 & -2\end{bmatrix} \text{\_\_\_\_\_\_\_\_\_\_ (i)}$$

$$(AdjA)A=\begin{bmatrix}6 & -2\\-4 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\4 & 6\end{bmatrix}$$

$$(AdjA)A=\begin{bmatrix}(6)\times 12\times 4 & (6)\times 2+(-2)\times 6\\(-4)\times 1+(1)(4) & (-4)(2)+(1)(6)\end{bmatrix}$$

$$=\begin{bmatrix}6-8 & 12-12\\-4+4 & -8+6\end{bmatrix}$$

$$\text{(AdjA)A}=\begin{bmatrix}-2 & 0\\0 & -2\end{bmatrix} \text{\_\_\_\_\_\_\_\_\_\_ (ii)}$$

$$(\det A)\text{I}=-2\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}-2\times 1 & 0\times 2\\-2\times 0 & 1\times -2\end{bmatrix}$$

$$\text{(detA)I}=\begin{bmatrix}-2 & 0\\0 & -2\end{bmatrix} \text{\_\_\_\_\_\_\_\_\_\_ (iii)}$$

Hence proved

From eq (i), (ii) and (iii)

A(AdjA)=(AdjA)A=(detA)I

(ii) $BB^{-1} = I = B^{-1}B$

Solution: $BB^{-1} = I = B^{-1}B$

To write in determinant form

$$|B|=\begin{vmatrix}3 & -1\\2 & -2\end{vmatrix}$$

$=-6-(-2)$

$=-6+2$

$=-4\neq 0$ (None singular)

$=B^{-1}$ exists.

To write in AdjB

$$AdjB=\begin{bmatrix}-2 & 1\\-2 & 3\end{bmatrix}$$

$$B^{-1}\frac{1}{|B|}Adj$$

$$B^{-1}=\frac{1}{-4}\begin{bmatrix}-2 & 1\\-2 & 3\end{bmatrix}$$

New

$$BB^{-1}=\begin{bmatrix}3 & -1\\2 & -2\end{bmatrix}\times\frac{1}{-4}\begin{bmatrix}-2 & 1\\-2 & 3\end{bmatrix}$$

$$=\frac{1}{-4}\begin{bmatrix}3 & -1\\2 & -2\end{bmatrix}\begin{bmatrix}-2 & 1\\-2 & 3\end{bmatrix}$$

$$=-\frac{1}{4}\begin{bmatrix}-6+2 & 3-3\\-4+4 & 2-6\end{bmatrix}$$

$$=\frac{1}{-4}\begin{bmatrix}-4 & 0\\0 & -4\end{bmatrix}$$

$$=\begin{bmatrix}\frac{4}{4} & 0\\0 & \frac{4}{4}\end{bmatrix}$$

$$BB^{-1}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=I$$

and

$$B^{-1}B=\frac{1}{-4}\begin{bmatrix}-2 & 1\\-2 & 3\end{bmatrix}\begin{bmatrix}3 & -1\\2 & -2\end{bmatrix}$$

$$=\frac{1}{-4}\begin{bmatrix}-6+2 & 2-2\\-6+6 & 2-6\end{bmatrix}$$

$$=\frac{1}{-4}\begin{bmatrix}-4 & 0\\0 & -4\end{bmatrix}$$

$$=\begin{bmatrix}\frac{-4}{-4} & 0\\ 0 & \frac{-4}{-4}\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$B^{-1}B \;=I$

From (i) and (ii)

$BB^{-1}=I=B^{-1}B$

Hence proved

Q.5 Determine whether the given matrices are multiplicative inverses of each other.

(i) $\begin{bmatrix}3 & 5\\ 4 & 7\end{bmatrix}$ and $\begin{bmatrix}7 & -5\\ -4 & 3\end{bmatrix}$

Solution: $\begin{bmatrix}3 & 5\\ 4 & 7\end{bmatrix}$ and $\begin{bmatrix}7 & -5\\ -4 & 3\end{bmatrix}$

$$\begin{bmatrix}3 & 5\\ 4 & 7\end{bmatrix}\begin{bmatrix}7 & -5\\ -4 & 3\end{bmatrix}$$

$$=\begin{bmatrix}21+(-20) & -15+15\\ 28+(-28) & -20+21\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

The given matrices are multiplicative inverse of each other.

(ii) $\begin{bmatrix}1 & 2\\ 2 & 3\end{bmatrix}$ and $\begin{bmatrix}-3 & 2\\ 2 & -1\end{bmatrix}$

Solution: $\begin{bmatrix}1 & 2\\ 2 & 3\end{bmatrix}$ and $\begin{bmatrix}-3 & 2\\ 2 & -1\end{bmatrix}$

$$\begin{bmatrix}1 & 2\\ 2 & 3\end{bmatrix}\begin{bmatrix}-3 & 2\\ 2 & -1\end{bmatrix}$$

$$=\begin{bmatrix}1 & 2\\ 2 & 3\end{bmatrix}\begin{bmatrix}-3 & 2\\ 2 & -1\end{bmatrix}$$

$$=\begin{bmatrix}-3+4 & 2+(-2)\\ -6+6 & 4+(-3)\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Given matrices are multiplicative inverse of each other

Q.6

(i) $(AB)^{-1}=B^{-1}A^{-1}$

Solution: $(AB)^{-1}=B^{-1}A^{-1}$

$$A=\begin{bmatrix}4 & 0\\ -1 & 2\end{bmatrix}, B=\begin{bmatrix}-4 & -2\\ 1 & -1\end{bmatrix}$$

$$AB=\begin{bmatrix}4 & 0\\ -1 & 2\end{bmatrix}\begin{bmatrix}-4 & -2\\ 1 & -1\end{bmatrix}$$

$$=\begin{bmatrix}4\times(-4)+0(1) & 4\times(-2)+0(-1)\\ -1\times(-4)+2(1) & -1\times(-2)+2(-1)\end{bmatrix}$$

$$=\begin{bmatrix}-16+0 & -8+0\\ 4+2 & 2+(-2)\end{bmatrix}$$

$$=\begin{bmatrix}-16 & -8\\ 6 & 0\end{bmatrix}$$

To write in determinant form

$$|AB|=\begin{vmatrix}-16 & -8\\ 6 & 0\end{vmatrix}$$

$$|AB|=0-(-48)$$

$$|AB|=48$$

To write in Adj (AB)

$$\text{Adj (AB)} = \begin{bmatrix}0 & 8\\ -6 & -16\end{bmatrix}$$

$$(AB)^{-1}=\frac{1}{|AB|}\times AdjAB$$

$$=\frac{1}{48}\times\begin{bmatrix}0 & 8\\ -6 & -16\end{bmatrix}$$

$$=\begin{bmatrix}\frac{0}{48} & \frac{8}{48}\\ \frac{-6}{48} & \frac{-16}{48}\end{bmatrix}$$

$$=\begin{bmatrix}0 & \frac{1}{6}\\ -\frac{1}{8} & -\frac{1}{3}\end{bmatrix}$$

To solve R.H. S

To write in determinant form

$$|B|=\begin{vmatrix}-4 & -2\\ 1 & -1\end{vmatrix}$$

$|B| = 4-(-2)$

$|B| = 4+2$

$|B| = 6$

To write in Adj B

$AdjB = \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}$

$B^{-1} = \frac{1}{|B|} \times AdjB$

By putting value

$B^{-1} = \frac{1}{6} \times \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}$

To write in determinant form

$|A| = \begin{vmatrix} 4 & 0 \\ -1 & 2 \end{vmatrix}$

$|A| = 8-(-0)$

$|A| = 8$

To write in Adj A

$AdjA = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \times AdjA$

$= \frac{1}{8} \times \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$

To solve R.H.S

$B^{-1}A^{-1} = \frac{1}{6}\begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix} \times \frac{1}{8}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$

$= \frac{1}{6} \times \frac{1}{8}\begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$

$= \frac{1}{48}\begin{bmatrix} -2+2 & 0+8 \\ -2-4 & 0-16 \end{bmatrix}$

$= \frac{1}{48}\begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}$

$= \begin{bmatrix} \frac{0}{48} & \frac{8}{48} \\ \frac{-6}{48} & \frac{-16}{48} \end{bmatrix}$

$= \begin{bmatrix} 0 & \frac{1}{6} \\ -\frac{1}{8} & -\frac{1}{3} \end{bmatrix}$

$= \begin{bmatrix} 0 \times \frac{1}{48} & 8 \times \frac{1}{48} \\ -6 \times \frac{1}{48} & -16 \times \frac{1}{48} \end{bmatrix}$

$= \begin{bmatrix} 0 & \frac{1}{6} \\ -\frac{1}{8} & -\frac{1}{3} \end{bmatrix}$

Hence proved

L.H.S = R.H.S

Last Updated: September 2020

Report any mistake at freeilm786@gmail.com

Exercise 1.6

Q.1 Use of matrices, if possible to solve the following systems of linear equations.

(i) The matrices inversion method

(ii) The Cramer's rule

(i) $2x-2y=4$

$3x+2y=6$

By matrices inversion method

$$\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

Let $A=\begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix} X=\begin{bmatrix} x \\ y \end{bmatrix} B=\begin{bmatrix} 4 \\ 6 \end{bmatrix}$

$$A=\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$$

$$|A|=\begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix}$$

$$|A|=(2)(2)-(-2)(3)$$

$$|A|=4+6$$

$$|A|=10$$

Then, solution is possible because A is non-singular matrix.

$$AdjA \ =\begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}$$

As we know that

$$AX=B$$

$$X=A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\frac{1}{|A|}\times AdjA\times B$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\frac{1}{10}\begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$=\frac{1}{10}\begin{bmatrix} 2\times 4 \ +2\times 6 \\ -3\times 4+2\times 6 \end{bmatrix}$$

$$=\frac{1}{10}\begin{bmatrix} 8+12 \\ -12+12 \end{bmatrix}$$

$$=\frac{1}{10}\begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} \frac{20}{10} \\ \frac{0}{10} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$x=2, y=0$$

Solution Set = {(2, 0)}

By Cramer's rule

$A=\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix} B=\begin{bmatrix} 4 \\ 6 \end{bmatrix}$

$$|A|=\begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix}$$

$$=(2)(2)-(-2)(3)$$

$$=4-(-6)$$

$$=4+6$$

$$=10$$

$$|A_x|=\begin{vmatrix} 4 & -2 \\ 6 & 2 \end{vmatrix}$$

$$=(4)(2)-(-2)(6)$$

$$=8+12$$

$$=20$$

$$|A_y|=\begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix}$$

$$=(2)(6)-(4)(3)$$

$$=12-12$$

$$=0$$

$$x=\frac{|A_x|}{|A|}$$

$$x=\frac{20}{10}$$

$$x=2$$

$$y=\frac{|A_y|}{|A|}$$

$y = \frac{0}{10}$

$y = 0$

Solution Set = {(2, 0)}

(ii) $2x + y = 3$

$6x + 5y = 1$

Matrices inversion method

$$\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Let $A = \begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix} X = \begin{bmatrix} x \\ y \end{bmatrix} B = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$

$$A = \begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 1 \\ 6 & 5 \end{vmatrix}$$

$= (2)(5) - (1)(6)$

$= 10 - 6$

$= 4$

Solution is possible because A is non-singular matrix.

$$AdjA = \begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix}$$

$AX = B$

$X = A^{-1}B$

$$X = \frac{1}{|A|} \times AdjA \times B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 5 \times 3 + (-1 \times 1) \\ -6 \times 3 + 2 \times 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 15 + (-1) \\ -18 + 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 14 \\ -16 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{14}{4} \\ \frac{16}{4} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{7}{2} \\ -4 \end{bmatrix}$$

$x = \frac{7}{2}, y = -4$

Solution Set = $\{(\frac{7}{2}, -4)\}$

By Cramer's Rule

$$A = \begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix} B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 1 \\ 6 & 5 \end{vmatrix}$$

$= (2)(5) - (1)(6)$

$= 10 - 6$

$= 4$

Solution is possible because A is non-singular matrix.

$$|A_x| = \begin{vmatrix} 3 & 1 \\ 1 & 5 \end{vmatrix}$$

$= (3)(5) - (1)(1)$

$= 15 - 1$

$= 14$

$$|A_y| = \begin{vmatrix} 2 & 3 \\ 6 & 1 \end{vmatrix}$$

$= (2)(1) - (3)(6)$

$= 2 - 18$

$= -16$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{14}{4}$$

$$x = \frac{7}{2}$$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{16}{4}$$

$y = -4$

Solution Set $= \left\{\left(\frac{7}{2}, -4\right)\right\}$

(iii) $4x+2y=8$
$3x-y=-1$

By Matrices Inversion Method

$$\begin{bmatrix}4 & 2\\3 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}8\\-1\end{bmatrix}$$

Let $A=\begin{bmatrix}3 & -2\\3 & 2\end{bmatrix}X=\begin{bmatrix}x\\y\end{bmatrix}B=\begin{bmatrix}4\\6\end{bmatrix}$

$$|A|=\begin{vmatrix}4 & 2\\3 & -1\end{vmatrix}$$
$=(4)(-1)-(2)(3)$
$=-4-6$
$=-10$

Solution is possible because A is non singular matrix.

$$AdjA \quad =\begin{bmatrix}-1 & -2\\-3 & 4\end{bmatrix}$$

As we know that
$AX=B$
$X=A^{-1}B$
$X=A^{-1}B$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{|A|}\times AdjA\times B$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{-10}\begin{bmatrix}-1 & -2\\-3 & 4\end{bmatrix}\begin{bmatrix}8\\-1\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{-10}\begin{bmatrix}-8+2\\-24+(-4)\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{-10}\begin{bmatrix}-6\\-28\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\frac{-6}{-10}\\\frac{-28}{-10}\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\frac{3}{5}\\\frac{14}{5}\end{bmatrix}$$
$$x=\frac{3}{5},y=\frac{14}{5}$$

Solution Set $=\left\{\left(\frac{3}{5},\frac{14}{5}\right)\right\}$

By Cramer's rule

$$A=\begin{bmatrix}4 & 2\\3 & -1\end{bmatrix}B=\begin{bmatrix}8\\-1\end{bmatrix}$$
$$|A|=\begin{vmatrix}4 & 2\\3 & -1\end{vmatrix}$$
$=(4)(-1)-(2)(3)$
$=-4-6$
$=-10$

Solution is possible because A is non singular matrix.

$$|A_x|=\begin{vmatrix}8 & 2\\-1 & -1\end{vmatrix}$$
$=(8)(-1)-(2)(-1)$
$=-8-(-2)$
$=-6$

$$x=\frac{|A_x|}{|A|}$$
$$x=\frac{6}{10}$$
$$x=\frac{3}{5}$$

$$|A_y|=\begin{bmatrix}4 & 8\\3 & -1\end{bmatrix}$$
$=(4)(-1)-(8)(3)$
$=-4-24$
$=-28$

$$y=\frac{|A_y|}{|A|}$$
$$y=\frac{28}{10}$$
$$y=\frac{14}{5}$$

Solution Set $=\left\{\left(\frac{3}{5},\frac{14}{5}\right)\right\}$

(iv) $3x$-$2y$=-6
$5x$-$2y$=-10

By Matrices Inversion Method

$$\begin{bmatrix}3 & -2\\5 & -2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-6\\-10\end{bmatrix}$$

Let $A=\begin{bmatrix}3 & -2\\3 & 2\end{bmatrix}X=\begin{bmatrix}x\\y\end{bmatrix}B=\begin{bmatrix}4\\6\end{bmatrix}$

$$|A|=\begin{vmatrix}3 & -2\\ 5 & -2\end{vmatrix}$$
$$=(3)(-2)-(-2)(5)$$
$$=-6-(-10)$$
$$=-6+10$$
$$=4$$

Solution is possible because A is non singular matrix.

$$AdjA=\begin{bmatrix}-2 & 2\\ -5 & 3\end{bmatrix}$$
$$X=A^{-1}B$$
$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{|A|}\times AdjA\times B$$
$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}-2 & 2\\ -5 & 3\end{bmatrix}\begin{bmatrix}-6\\ -10\end{bmatrix}$$
$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}-2\times-6+2\times-10\\ -5\times-63\times-10\end{bmatrix}$$
$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}12+(-20)\\ 30+(-30)\end{bmatrix}$$
$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}12-20\\ 30-30\end{bmatrix}$$
$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}-8\\ 0\end{bmatrix}$$
$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}\frac{-8}{4}\\ \frac{0}{4}\end{bmatrix}$$
$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}-2\\ 0\end{bmatrix}$$

$x=-2$, $y=0$

Solution Set $=\{(-2,0)\}$

By Cramer's rule

$$A=\begin{bmatrix}3 & -2\\ 5 & -2\end{bmatrix}, B=\begin{bmatrix}-6\\ -10\end{bmatrix}$$
$$|A|=\begin{vmatrix}3 & -2\\ 5 & -2\end{vmatrix}$$
$$=(3)(-2)-(-2)(5)$$
$$=-6-(-10)$$
$$=-6+10$$
$$=4$$

Solution is possible because A is non singular matrix.

$$|A_x|=\begin{vmatrix}-6 & -2\\ -10 & -2\end{vmatrix}$$
$$=(-6)(-2)-(-2)(-10)$$
$$=+12-(+20)$$
$$=12-20$$
$$=-8$$
$$|A_y|=\begin{vmatrix}3 & -6\\ 5 & -10\end{vmatrix}$$
$$=(3)(-10)-(-6)(5)$$
$$=-30-(-30)$$
$$=-30+30$$
$$=0$$
$$x=\frac{|A_x|}{|A|}$$
$$x=\frac{-8}{4}$$
$$x=-2$$
$$y=\frac{|A_y|}{|A|}$$
$$=\frac{0}{4}$$
$$y=0$$

Solution Set $=\{(-2,0)\}$

(v) $3x-2y=4$
$-6x+4y=7$

By Matrices Inversion Method

$$\begin{bmatrix}3 & -2\\ -6 & 4\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}4\\ 7\end{bmatrix}$$
$$A=\begin{bmatrix}3 & -2\\ 3 & 2\end{bmatrix} X=\begin{bmatrix}x\\ y\end{bmatrix} B=\begin{bmatrix}4\\ 6\end{bmatrix}$$
$$|A|=\begin{vmatrix}3 & -2\\ -6 & 4\end{vmatrix}$$
$$=(3)(4)-(-2)(-6)$$
$$=12-(+12)$$
$$=12-12$$
$$=0$$

Solution is not possible because A is singular matrix.

(vi) $4x+y=9$
$-3x-y=-5$

By Matrices Inversion Method

$$\begin{bmatrix}4 & 1\\-3 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}9\\-5\end{bmatrix}$$

Let $A=\begin{bmatrix}3 & -2\\3 & 2\end{bmatrix} X=\begin{bmatrix}x\\y\end{bmatrix} B=\begin{bmatrix}4\\6\end{bmatrix}$

$$|A|=\begin{vmatrix}4 & 1\\-3 & -1\end{vmatrix}$$
$=(4)(-1)-(-1)(-3)$
$=-4+3$
$=-1$

Solution is possible because $|A|$ is non singular

$$AdjA=\begin{vmatrix}-1 & -1\\3 & 4\end{vmatrix}$$

As we know that

$X=A^{-1}B$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{|A|}\times AdjA\times B$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{-1}\begin{bmatrix}-1 & -1\\3 & 4\end{bmatrix}\begin{bmatrix}9\\-5\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{-1}\begin{bmatrix}-9+5\\27+(-20)\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{-1}\begin{bmatrix}-4\\7\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\frac{-4}{-1}\\\frac{7}{-1}\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}4\\-7\end{bmatrix}$$

$x=4, y=-7$

Solution Set$=\{(4,-7)\}$

By Cramer's rule

$A=\begin{bmatrix}4 & 1\\-3 & -1\end{bmatrix}, B=\begin{bmatrix}9\\-5\end{bmatrix}$

$$|A|=\begin{vmatrix}4 & 1\\-3 & -1\end{vmatrix}$$
$=(4)(-1)-(1)(-3)$
$=-4-(-3)$
$=-4+3$
$=-1$

$$|A_x|=\begin{vmatrix}9 & 1\\-5 & -1\end{vmatrix}$$
$=(9)(-1)-(1)(-3)$
$=-9-(-5)$
$=-9+5$
$=-4$

$x=\frac{|A_x|}{|A|}$

$=\frac{-4}{-1}$

$x=4$

$$|A_y|=\begin{bmatrix}4 & 9\\-3 & -5\end{bmatrix}$$
$=(4)(-5)-(9)(-3)$
$=-20-(-27)$
$=--20+27$
$=7$

$y=\frac{|A_y|}{|A|}$

$=\frac{7}{-1}$

$y=-7$

Solution Set$=\{(4_7,-7)\}$

(vii) $2x-2y=4$
$-5x-2y=-10$

By Matrices Inversion Method

$$\begin{bmatrix}2 & -2\\-5 & -2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}4\\-10\end{bmatrix}$$

Let $A=\begin{bmatrix}3 & -2\\3 & 2\end{bmatrix} X=\begin{bmatrix}x\\y\end{bmatrix} B=\begin{bmatrix}4\\6\end{bmatrix}$

$$|A|=\begin{vmatrix}2 & -2\\-5 & -2\end{vmatrix}$$
$=(2)(-2)-(-2)(-5)$
$=-4-(+10)$
$=-4-10$
$=-14$

Solution is possible

$AdjA = \begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix}$

As we know that

$X = A^{-1}B$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|A|} \times AdjA \times B$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-14}\begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ -10 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-14}\begin{bmatrix} -8+(-20) \\ 20+(-20) \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-14}\begin{bmatrix} -8-20 \\ 20-20 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-14}\begin{bmatrix} -28 \\ 0 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-28}{-14} \\ \frac{0}{14} \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$x = 2, y = 0$

Solution Set $= \{(2,0)\}$

By Cramer's rule

$A = \begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix} B = \begin{bmatrix} 4 \\ -10 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & -2 \\ -5 & -2 \end{vmatrix}$

$= (2)(-2) - (-2)(-5)$

$= -4 - (+10)$

$= -4 - 10$

$= -14$

Set is possible

$|A_x| = \begin{bmatrix} 4 & -2 \\ -10 & -2 \end{bmatrix}$

$= (4)(-2) - (-2)(-10)$

$= -8 - (+20)$

$= -8 - 20$

$= -28$

$|A_y| = \begin{bmatrix} 2 & 4 \\ -5 & -10 \end{bmatrix}$

$= (2)(-10) - (4)(-5)$

$= -20 - (-20)$

$= -20 + 20$

$= 0$

$x = \frac{|A_x|}{|A|}$

$x = \frac{-28}{-14}$

$x = 2$

Solution Set $= \{(2,0)\}$

(viii) $3x - 4y = 4$

$x + 2y = 8$

By Matrices Inversion Method

$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$

Let $A = \begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix} X = \begin{bmatrix} x \\ y \end{bmatrix} B = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$

$|A| = \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix}$

$= (3)(2) - (-4)(1)$

$= 6 - (-4)$

$|A| = 6 + 4$

$= 10$

Solution is possible because A is non singular matrix.

$AdjA = \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}$

As we know that

$AX = B$

$X = A^{-1}B$

$X = \frac{1}{|A|} \times AdjA \times B$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 2\times4+4\times8 \\ -1\times+3\times8 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 8+32 \\ -4+24 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 40 \\ 20 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{40}{10} \\ \frac{20}{10} \end{bmatrix}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$x = 4, y = 2$

Solution Set $= \{(4,2)\}$

By Cramer's rule

$$A = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} B = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix}$$

$= (3)(2) - (-4)(1)$

$= 6 - (-4)$

$= 6 + 4$

$= 10$

Solution is possible

$$|A_x| = \begin{bmatrix} 4 & -4 \\ 8 & 2 \end{bmatrix}$$

$$|A_x| = \begin{vmatrix} 4 & -4 \\ 8 & 2 \end{vmatrix}$$

$= (4)(2) - (-4)(8)$

$= 8 - (-32)$

$= 8 + 32$

$= 40$

$$|A_y| = \begin{bmatrix} 3 & 4 \\ 1 & 8 \end{bmatrix}$$

$$|A_y| = \begin{vmatrix} 3 & 4 \\ 1 & 8 \end{vmatrix}$$

$= (3)(8) - (4)(1)$

$= 24 - 4$

$= 20$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{40}{10}$$

$x = 4$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{20}{10}$$

$y = 2$

Solution Set $= \{(4,2)\}$

Q.2 The length of a rectangle is 4 times it width. The perimeter of the rectangle is 150cm. Find the dimensions of the rectangle.

Solution:

Let width of rectangle $= x$

Length of rectangle $= y$

According to 1st condition

$y = 4x$

$-4x + y = 0 \qquad \rightarrow ...(i)$

According to 2nd condition

2(length + Width)=Perimeter

$2(y + x) = 150$

$$y + x = \frac{\cancel{150}^{75}}{\cancel{2}}$$

$x + y = 75 \qquad \rightarrow ...(ii)$

$-4x + y = 0$

$x + y = 75$

Changing into matrix form

$$\begin{bmatrix} -4 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

$X = A^{-1}B$

(By matrix inversion method)

Let $A = \begin{bmatrix} -4 & 1 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} x \\ y \end{bmatrix} B = \begin{bmatrix} 0 \\ 75 \end{bmatrix}$

$$|A| = \begin{vmatrix} -4 & 1 \\ 1 & 1 \end{vmatrix}$$

$= (-4)(1) - (1)(1)$

$= -4 - 1$

$= -5$

$$AdjA = \begin{bmatrix} 1 & -1 \\ -1 & -4 \end{bmatrix}$$

As we know that

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|A|} \times AdjA \times B$$

$$= -\frac{1}{5} \begin{bmatrix} 1 & -1 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

$$= -\frac{1}{5} \begin{bmatrix} 0 - 75 \\ 0 - 300 \end{bmatrix}$$

$$= \frac{1}{-5} \begin{bmatrix} -75 \\ -300 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-75}{-5} \\ \frac{-300}{-5} \end{bmatrix}$$

$$= \begin{bmatrix} 15 \\ 60 \end{bmatrix}$$

$x = 15, y = 60$

Width of rectangle = x = 15cm

Length of rectangle = y = 60cm

By Cramer's rule

$$A = \begin{bmatrix} -4 & 1 \\ 1 & 1 \end{bmatrix} B = \begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

$$|A| = \begin{vmatrix} -4 & 1 \\ 1 & 1 \end{vmatrix}$$

$= (-4)(1) - (1)(1)$

$= -4 - 1$

$= -5$

$$|A_x| = \begin{vmatrix} 0 & 1 \\ 75 & 1 \end{vmatrix}$$

$= (0)(1) - (1)(75)$

$= 0 - 75$

$= -75$

$$|A_y| = \begin{vmatrix} -4 & 0 \\ 1 & 75 \end{vmatrix}$$

$= (-4)(75) - (0)(1)$

$= 0 - 300$

$= -300$

$$x = \frac{|A_x|}{|A|}$$

$$= \frac{-75}{-5}$$

$x = 15$

$$y = \frac{|A_y|}{|A|}$$

$$= \frac{-300}{-5}$$

$y = 60$

Then

Width of rectangle = x = 15 cm

Length of rectangle = y = 60 cm

Q.3 Two sides of a rectangle differ by 3.5cm. Find the dimension of the rectangle if its perimeter is 67cm.

Solution:

Suppose Width of rectangle = x

Length of rectangle = y

According to 1st condition

$y - x = 3.5$

$-x + y = 3.5 \quad \rightarrow \ldots(\text{i})$

According to 2nd condition

$2(L + B) = P$

$2(y + x) = 67$

$$x + y = \frac{67}{2}$$

$x + y = 33.5 \quad \rightarrow (\text{ii})$

Changing into matrix form

$$\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3.5 \\ 33.5 \end{bmatrix}$$

(By matrix inversion method)

Let $A = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} x \\ y \end{bmatrix} B = \begin{bmatrix} 3.5 \\ 33.5 \end{bmatrix}$

$$|A| = \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}$$

$= (-1)(1) - (1)(1)$

$= -1 - 1$

$= -2$

$$AdjA = \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$$

As we know that

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|A|} \times AdjA \times B$$

$$= \frac{1}{-2} \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 3.5 \\ +33.5 \end{bmatrix}$$

$$= \frac{1}{-2} \begin{bmatrix} 1\times3.5 & 1\times33.5 \\ -1\times3.5 & 1\times33.5 \end{bmatrix}$$

$$= \frac{1}{-2} \begin{bmatrix} 3.5(-33.5) \\ -3.5(-33.5) \end{bmatrix}$$

$$= \frac{1}{-2} \begin{bmatrix} -30 \\ -37 \end{bmatrix}$$

$$=\begin{bmatrix}\frac{30}{2}\\\frac{37}{2}\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}15\\\frac{37}{2}\end{bmatrix}$$

$$x=15, y=\frac{37}{2}=18.5$$

By Cramer's rule

$$A=\begin{bmatrix}-1 & 1\\1 & 1\end{bmatrix} B=\begin{bmatrix}3.5\\33.5\end{bmatrix}$$

$$|A|=\begin{vmatrix}-1 & 1\\1 & 1\end{vmatrix}$$

$$=(-1)(1)-(1)(1)$$
$$=-1-1$$
$$=-2$$

$$|A_x|=\begin{vmatrix}3.5 & 1\\33.5 & 1\end{vmatrix}$$

$$=(3.5)(1)-(1)(33.5)$$
$$=3.5-33.5$$
$$=-30$$

$$|A_y|=\begin{vmatrix}-1 & 3.5\\1 & 33.5\end{vmatrix}$$

$$=(-1)(33.5)-(3.5)(1)$$
$$=-33.5-3.5$$
$$=-37$$

$$x=\frac{|A_x|}{|A|}$$

$$x=\frac{-30}{-2}$$

$$x=15$$

$$y=\frac{|A_y|}{|A|}$$

$$y=\frac{-37}{-2}$$

$$y=\frac{37}{2}=18.5$$

Width of rectangle =x=15cm
Length of rectangle =y=18.5cm

Q.4 The third angle of an isosceles △ is $16°$ less than the sum of two equal angles. Find three angles of the triangle.

Solution:

Let each equal angles are x and third angle is y

According to condition $y=2x-16$

$2x-y=16$ (i)

As we know that

$x+x+y=180$

$2x+y=180$ (ii)

$2x-y=16$

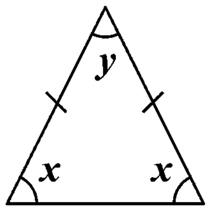
$2x+y=180$

Changing into matrix form

$$\begin{bmatrix}2 & -1\\2 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}16\\180\end{bmatrix}$$

$$X=A^{-1}B$$

Let $A=\begin{bmatrix}2 & -1\\2 & 1\end{bmatrix} X=\begin{bmatrix}x\\y\end{bmatrix} B=\begin{bmatrix}16\\180\end{bmatrix}$



$X=A^{-1}B$

Where

$$A=\begin{bmatrix}2 & -1\\2 & 1\end{bmatrix}, X=\begin{bmatrix}x\\y\end{bmatrix}, B=\begin{bmatrix}16\\180\end{bmatrix}$$

$$|A|=\begin{vmatrix}2 & -1\\2 & 1\end{vmatrix}$$

$$=2\times1-(-1)\times2$$
$$=2+2$$

$=4\neq0$ (None singular)

A^{-1} exist

$$AdjA=\begin{bmatrix}1 & 1\\-2 & 2\end{bmatrix}$$

$$X=A^{-1}B$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}1 & 1\\-2 & 2\end{bmatrix}\begin{bmatrix}16\\180\end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix}1\times16+1\times180\\-2\times16+2\times180\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}16+180\\-32+360\end{bmatrix}$$

$$=\begin{bmatrix}196\\328\end{bmatrix}$$

$$=\begin{bmatrix}\frac{196}{4}\\\frac{328}{4}\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}49\\82\end{bmatrix}$$

$x=49$

$y=82$

Cramer Rule

$$A=\begin{bmatrix}2&-1\\2&1\end{bmatrix}B=\begin{bmatrix}16\\180\end{bmatrix}$$

$$|A|=\begin{vmatrix}2&-1\\2&1\end{vmatrix}$$

$=(2)(1)-(-1)(2)$

$=2-(-2)$

$=2+2$

$=4$

$$|A_x|=\begin{bmatrix}16&-1\\180&1\end{bmatrix}$$

$=(16)(1)-(-1)(180)$

$=16+180$

$=196$

$$|A_y|=\begin{bmatrix}2&16\\2&180\end{bmatrix}$$

$=(2)(180)-(16)(2)$

$=360-32$

$=328$

$$x=\frac{|A_x|}{|A|}$$

$$x=\frac{196}{4}$$

$x=49$

$$y=\frac{|A_y|}{|A|}$$

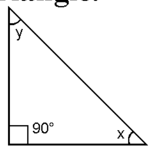
$$y=\frac{328}{4}$$

$y=82$

1st angle = x = 49° Ans
2nd angle= x = 49° Ans
3rd angle =y= 82° Ans

Q.5 One acute angle of a right triangle is $12°$ more than twice the other acute angle. Find the acute angles of the right triangle.

Solution:



Let one acute angle =x
And other acute angle =y
According to 1st condition

$x=2y+12$

$x-2y=12 \quad \rightarrow(\text{i})$

As we know

$x+y=90 \quad \rightarrow(\text{ii})$

By matrices inversion method
Changing into matrix form

$$\begin{bmatrix}1&-2\\1&1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}12\\90\end{bmatrix}$$

$X=A^{-1}B$

Let $A=\begin{bmatrix}1&-2\\1&1\end{bmatrix}X=\begin{bmatrix}x\\y\end{bmatrix}B=\begin{bmatrix}12\\90\end{bmatrix}$

$$|A|=\begin{vmatrix}1&-2\\1&1\end{vmatrix}$$

$=(1)(1)-(-2)(1)$

$=1-(-2)$

$=3$ (Non singular)

$\therefore A^{-1}$ exists

$$AdjA=\begin{bmatrix}1&2\\-1&1\end{bmatrix}$$

As we know that

$X=A^{-1}B$ or

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{|A|}\times AdjA\times B$$

$$=\frac{1}{3}\begin{bmatrix}1&2\\-1&1\end{bmatrix}\begin{bmatrix}12\\90\end{bmatrix}$$

$$=\frac{1}{3}\begin{bmatrix}12+180\\-12+90\end{bmatrix}$$

$$=\frac{1}{3}\begin{bmatrix}192\\78\end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{192}{3} \\ \frac{78}{3} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 64 \\ 26 \end{bmatrix}$$

$x = 64, y = 26$

Then

1^{st} angle = x = 64°

2^{nd} angle = y = 26°

By Cramer's rule

$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} B = \begin{bmatrix} 12 \\ 90 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix}$

$= (1)(1) - (-2)(1)$

$= 1 - (-2)$

$= 1 + 2$

$= 3$

$|A_x| = \begin{vmatrix} 12 & -2 \\ 90 & 1 \end{vmatrix}$

$= (12)(1) - (-2)(90)$

$= 12 + 180$

$= 192$

$|A_y| = \begin{vmatrix} 1 & 12 \\ 1 & 90 \end{vmatrix}$

$= (90) - (12)$

$= 90 - 12$

$= 78$

$x = \frac{|A_x|}{|A|}$

$x = \frac{192}{3}$

$x = 64$

$y = \frac{|A_y|}{|A|}$

$y = \frac{78}{3}$

$y = 26$

1^{st} angle = x = 64°

2^{nd} angle = y = 26°

Q.6 Two cars that are 600 km apart are moving towards each other. Their speeds differ by 6 km per hour and the cars are 123 km apart after $4\frac{1}{2}$ hours. Find the speed of each car.

Solution:

Suppose speed of 1^{st} car = x

Suppose speed of 2^{nd} car = y

According to 1^{st} condition

$x - y = 6 \qquad \rightarrow (i)$

According to 2^{nd} condition

Total distance = 600 km

Left distance = 123 km

Covered distance = total distance-left distance

Covered distance = 600-123

= 477 km

Total time = $4\frac{1}{2}$ hours =or $\frac{9}{2}$ hours

$$\text{Total Speed} = \frac{\text{Total Distance Covered}}{\text{Total Time Taken}}$$

$x + y = \frac{477}{\frac{9}{2}} = 477 \div \frac{9}{2} = 477 \times \frac{2}{9}$

$x + y = \frac{^{53}\cancel{477} \times 2}{\cancel{9}}$

$x + y = 106 \qquad \rightarrow (ii)$

x - y = 6

x + y = 106

By matrices inversion method

Changing into matrix form

$$\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}6\\ 106\end{bmatrix}$$

$X = A^{-1}B$, where

Let $A=\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}6\\ 106\end{bmatrix}$

$$|A|=\begin{vmatrix}1 & -1\\ 1 & 1\end{vmatrix}$$

$$=(1)(1)-(-1)(1)$$

$$=1-(-1)$$

$$=1+1$$

$$=2$$

$$AdjA=\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}$$

$$X = A^{-1}B$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{|A|}\times AdjA\times B$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{2}\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}\begin{bmatrix}6\\ 106\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}6+106.\\ -6+106\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}112\\ 100\end{bmatrix}$$

$$=\begin{bmatrix}\frac{112}{2}\\ \frac{100}{2}\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}56\\ 50\end{bmatrix}$$

$x=56, y=50$

Speed of 1st car = x= 56km/h

Speed of 2nd car = y= 50km/h

By Cramer's rule

$$A=\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix} B=\begin{bmatrix}6\\ 106\end{bmatrix}$$

$$|A|=\begin{vmatrix}1 & -1\\ 1 & 1\end{vmatrix}$$

$$=(1)(1)-(-1)(1)$$

$$=1-(-1)$$

$$=1+1$$

$$=2$$

$$|A_x|=\begin{vmatrix}6 & -1\\ 106 & 1\end{vmatrix}$$

$$=(6)(1)-(-1)(106)$$

$$=6-(-106)$$

$$=6+106$$

$$=112$$

$$|A_y|=\begin{vmatrix}1 & 6\\ 1 & 106\end{vmatrix}$$

$$=(106)(1)-(6)(1)$$

$$=106-6$$

$$=100$$

$$x=\frac{|A_x|}{|A|}$$

$$x=\frac{112}{2}$$

$$x=56$$

$$y=\frac{|A_y|}{|A|}$$

$$y=\frac{100}{2}$$

$$y=50$$

Then

Speed of 1st car = x = 56km/h

Speed of 2nd car = y = 50km/h

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Exercise 2.1

Q.1 Identity which of the following are rational and irrational numbers?

(i)	$\sqrt{3}$	Irrational number
(ii)	$\frac{1}{6}$	Rational number
(iii)	π	Irrational number
(iv)	$\frac{15}{2}$	Rational number
(v)	7.25	Rational number
(vi)	$\sqrt{29}$	Irrational number

Q.2 Convert the following fractions into decimal fractions.

(i) $\frac{17}{25}$

Solution: $\frac{17}{25}$

$$\begin{array}{r} 0.68 \\ 25\overline{)\ 170} \\ -150 \\ \hline 200 \\ -200 \\ \hline 0 \end{array}$$

$\frac{17}{25} = 0.68$ **Ans**

(ii) $\frac{19}{4}$

Solution: $\frac{19}{4}$

$$\begin{array}{r} 4.75 \\ 4\overline{)19.000} \\ 16 \\ \hline 30 \\ 28 \\ \hline 20 \\ 20 \\ \hline 0 \end{array}$$

$= \frac{19}{4}$

$= 4.75$ **Ans**

(iii) $\frac{57}{8}$

Solution: $\frac{57}{8}$

$$\begin{array}{r} 7.125 \\ 8\overline{)\ 57} \\ -56 \\ \hline 10 \\ 8 \\ \hline 20 \\ -16 \\ \hline 40 \\ 40 \\ \hline 0 \end{array}$$

$= \frac{57}{8}$

$= 7.125$ **Ans**

(iv) $\frac{205}{18}$

Solution: $\frac{205}{18}$

$$\begin{array}{r} 11.388 \\ 18\overline{)205.000} \\ \hline 25 \end{array}$$

$$\begin{array}{r} 18 \\ \hline 70 \\ -54 \\ \hline 160 \\ -144 \\ \hline 160 \\ -144 \\ \hline 16 \end{array}$$

$\frac{208}{18}$

$= 11.3888$

$= 11.3889$ **Ans**

(v) $\frac{5}{8}$

Solution: $\frac{5}{8}$

$$\begin{array}{r} .625 \\ 8\overline{)5.000} \\ 48 \\ \hline 20 \\ -16 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

$\frac{5}{8}$

$= 0.625$ **Ans**

(vi) $\frac{25}{38}$

Solution: $\frac{25}{38}$

$$\begin{array}{r} 0.65789... \\ 38\overline{)\quad 250} \\ -228 \\ \hline 220 \\ -190 \\ \hline 300 \\ -266 \\ \hline 340 \\ -304 \\ \hline 360 \\ -342 \\ \hline 18 \end{array}$$

$\frac{25}{38}$

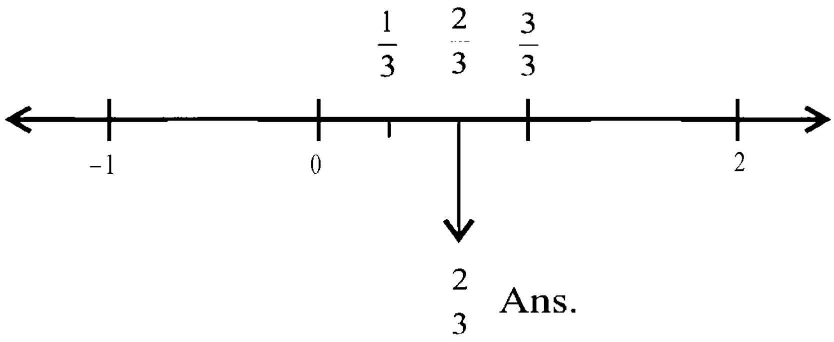
$= 0.65789$ **Ans**

Q.3 Which of the following statements are true and which are false?

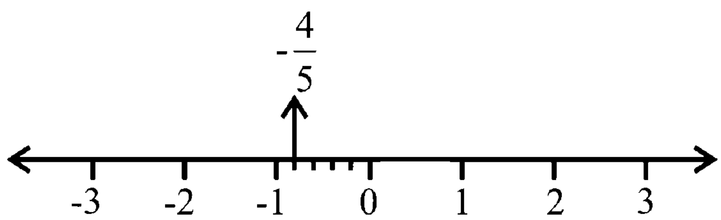
(i)	$\frac{2}{3}$ is an irrational number.	False
(ii)	π is an irrational number.	True
(iii)	$\frac{1}{9}$ is a terminating fraction.	False
(iv)	$\frac{3}{4}$ is a terminating fraction.	True
(v)	$\frac{4}{5}$ is a recurring fraction.	False

Q.4 Represent the following numbers on the number line.

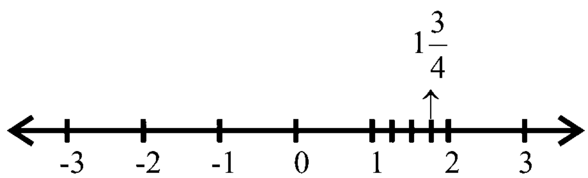
(i) $\frac{2}{3}$



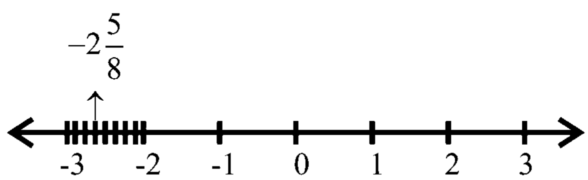
(ii) $-\frac{4}{5}$



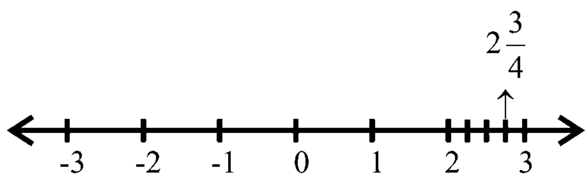
(iii) $1\frac{3}{4}$



(iv) $-2\frac{5}{8}$



(v) $2\frac{3}{4}$



(vi) $\sqrt{5}$

By Pythagoras theorem

$(\text{Hypoteneus})^2 = (\text{Base})^2 + (\text{Perpencicular})^2$

$\left(\overline{OB}\right)^2 = (2)^2 + (1)^2$

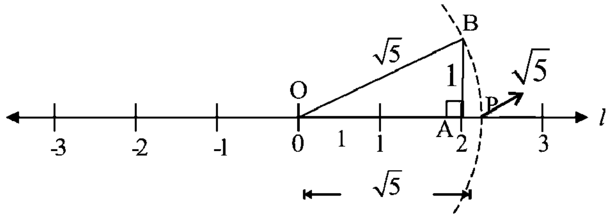
$\left(\overline{OB}\right)^2 = 4 + 1$

$\left(\overline{OB}\right)^2 = 5$

Taking square root on both sides

$\sqrt{\left(\overline{OB}\right)^2} = \sqrt{5}$

$\overline{OB} = \sqrt{5}$



Q.5 Give a rational number between $\frac{3}{4}$ and $\frac{5}{9}$

Solution:

Required No between

$\frac{3}{4}$ and $\frac{5}{9}$

$= \left[\frac{3}{4} + \frac{5}{9}\right] \div 2$

$= \left[\frac{27+20}{36}\right] \div 2$

$= \left[\frac{47}{36}\right] \div 2$

$= \frac{47}{36} \times \frac{1}{2}$

$= \frac{47}{72}$ **Ans**

Q.6 Express the following recurring decimals as the rational number $\frac{p}{q}$ where p, q are integer and $q \neq 0$.

(i) $0.\overline{5}$

Solution:

$x = 0.\overline{5}$

$x = 0.555...$

$10 \times x = 10 \times 0.555...$

$10x = 5.555...$

$10x = 5 + 0.555...$

$10x = 5 + x$

$10x - x = 5$

$9x = 5$

$x = \frac{5}{9}$

$\therefore 0.\overline{5} = \frac{5}{9}$ **Ans**

(ii) $0.\overline{13}$

Solutions:

Suppose

$x = 0.\overline{13}$

$x = 0.131313...$

$100^{x} x = 100 \times 1.131313...$

$100x = 13.1313...$

$100x = 13 + 0.1313...$

$100x = 13 + x$

$100x - x = 13$

$99x = 13$

$x = \dfrac{13}{99}$

$\therefore 0.\overline{13} = \dfrac{13}{99}$ **Ans**

(iii) $0.\overline{67}$

Solutions:

Suppose

$x = 0.\overline{67}$

$x = 0.676767...$

$100 \times x = 100 \times 0.676767...$

$100x = 67.6767...$

$100x = 67 + 0.6767...$

$100x = 67 + x$

$100x - x = 67$

$99x = 67$

$x = \dfrac{67}{99}$

$\therefore 0.\overline{67} = \dfrac{67}{99}$ **Ans**

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Exercise 2.2

Q.1 Identify the property used in the following.

(i)	$a+b=b+a$	Commutative Property *w.r.t* addition
(ii)	$(ab)c=a(bc)$	Associative Property *w.r.t* multiplication
(iii)	$7\times 1=7$	Multiplicative Identity
(iv)	$x>y$ or $x=y$ or $x<y$	Trichotomy
(v)	$ab=ba$	Commutative *w.r.t* multiplication
(vi)	$a+c=b+c=a+b$	Cancellation Property of addition
(vii)	$5+(-5)=0$	Additive Inverse
(viii)	$7\times\frac{1}{7}=1$	Multiplicative inverse
(ix)	$a>b\Rightarrow ac>bc(c>0)$	Multiplicative property

Q.2 Fill in the following blanks by stating the properties of real numbers used.

$3x+3(y-x)$

$=3x+3y-3x,...$ Distributive property

$=3x-3x+3y,...$ Commutative

$=0+3y,...$ Additive Inverse

$=3y,...$ Additive identity

Q.3 Give the name of property used in the following.

(i)	$\sqrt{24}+0=\sqrt{24}$	Additive Identity
(ii)	$-\frac{2}{3}\left[5+\frac{7}{2}\right]=\left[-\frac{2}{3}\right](5)+\left[-\frac{2}{3}\right]\left[\frac{7}{2}\right]$	Distributive Property
(iii)	$\pi+(-\pi)=0$	Additive Inverse
(iv)	$\sqrt{3}.\sqrt{3}$ is a real number.	Closure property w.r.t x.
(v)	$\left[-\frac{5}{8}\right]\left[-\frac{8}{5}\right]=1$	Multiplicative Inverse.

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Exercise 3.1

Q.1 Express each of the following numbers in scientific notations.

(i) 5700
$= 5.7\times10^{3}$ **Ans**

(ii) 49,800,000
$= 4.98\times10^{7}$ **Ans**

(iii) 96000000
$= 9.6\times10^{7}$ **Ans**

(iv) 416.9
$= 4.169\times10^{2}$ **Ans**

(v) 83000
$= 8.3\times10^{4}$ **Ans**

(vi) 0.00643
$= 6.43\times10^{-3}$ **Ans**

(vii) 0.0074
$= 7.4\times10^{-3}$ **Ans**

(viii) 60,000,000

$= 6\times10^{7}$ Ans

(ix) 0.00000000395
$= 3.95\times10^{-9}$ **Ans**

(x) $\dfrac{275000}{0.0025}$

$= \dfrac{2.75\times10^{5}}{2.5\times10^{-3}}$ **Ans**

Q.2 Express the following number in ordinary notation.

(i) 6×10^{-4}
$= 0.0006$ **Ans**

(ii) 5.06×10^{10}
$= 50600000000$ **Ans**

(iii) 9.018×10^{-6}
$= 0.000009018$ Ans

(iv) 7.865×10^{8}
$= 786500000$ **Ans**

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Exercise 3.2

Q.1 Find the common logarithms of each of the following numbers.

(i) 232.92

Solution: 232.92

Suppose $x = 232.92$

Taking log

$\log x = \log 232.92$

$Ch = 2$

Mantissa = 0.3672

$\log x = 2.3672$ **Ans**

(ii) 29.326

Solution: 29.326

Suppose $x = 29.326$

Taking log

$\log x = \log 29.326$

$Ch = 1$

Mantissa = 0.4672

$\log x = 1.4672$ **Ans**

(iii) 0.00032

Solution: 0.00032

Suppose $x = 0.00032$

Taking log

$\log x = \log 0.00032$

$Ch = \bar{4}$

Mantissa = 0.5051

$\log x = \bar{4}.5051$ **Ans**

(iv) 0.3206

Solution: 0.3206

Suppose $x = 0.3206$

Taking log:

$\log x = \log 0.3206$

$Ch = \bar{1}$

Mantissa = 0.5059

$\log x = \bar{1}.5059$ **Ans**

Q.2 If log $31.09 = 1.4926$, find the value of the following.

If

$\log 31.09 = 1.4926$

Then

(i) $\log 3.109 = 0.4926$

(ii) $\log 310.9 = 2.4926$

(iii) $\log 0.003109 = \bar{3}.4926$

(iv) $0.3109 = \bar{1}.4926$

Solution:

(i) $\log 3.109$

Characteristics = 0

Mantissa = 0.4926

$\log 3.109 = 0.4926$ **Ans**

(ii) $\log 310.9$

Characteristics = 2

Mantissa =0.4926

$\log 310.9 = 2.4926$ **Ans**

(iii) $\log 0.003109$

Characteristics = $\bar{3}$

Mantissa = 0.4926

$\log 0.003109 = \bar{3}.4926$ **Ans**

(iv) $\log 0.3109$

Characteristics = $\bar{1}$

Mantissa = 0.4926

$\log 0.3109 = \bar{1}.4926$ **Ans**

Q.3 Find the numbers whose common logarithms are

(i) 3.5621

Solution:

$\log x = 3.5621$

Ch=3(If ch is positive, then plus for reference point)

Mantissa = 0.5621

$x = \text{antilog}\, 3.5621$

$x = 3649.0$ **Ans**

(ii) $\bar{1}.7427$

Solution:

$\log x = \bar{1}.7427$

$\text{Ch} = \bar{1}$

Mantissa = 0.7427

$x = \text{anti}\log \bar{1}.7427$

$x = 0.5530$ **Ans**

Q.4 What replacement for the unknown in each of the following will make the true statements?

(i) $\log_3 81 = L$

Solution: $\log_3 81 = L$

Writing in exponential form.

$3^L = 81$

$3^L = 3^4$

$\because$ Bases are equal so

$L = 4$ **Ans**

(ii) $\log_a 6 = 0.5$

Solution: $\log_a 6 = 0.5$

$a^{0.5} = 6$

$a^{\frac{1}{2}} = 6$

$\sqrt{a} = 6$ Taking square on both sides

$\sqrt{(a)}^2 = (6)^2$

$a = 36$ **Ans**

(iii) $\log_5 n = 2$

Write in exponential form

$5^2 = n$

$25 = n$

Or $n = 25$ **Ans**

(iv) $10^P = 40$

Solution: $10^P = 40$

Changing into logarithmic form

$P = \log_{10} 40$

$= \log 40$

$= 1.6021$ **Ans**

Q.5 Evaluate.

(i) $\log_2 \frac{1}{128}$

Solution: $\log_2 \frac{1}{128}$

Suppose $\log_2 \frac{1}{128} = x$

Writing in exponential form.

$2^x = \frac{1}{128}$

$2^x = \frac{1}{2^7}$

$2^x = 2^{-7}$

$\because$ Bases are equal so

$x = -7$ **Ans**

(ii) log 512 to the base $2\sqrt{2}$

Solution: $\log_{2\sqrt{2}} 512 = x$

Writing in exponential form

$\left(2\sqrt{2}\right)^x = 512$

$\left(2^1 . 2^{\frac{1}{2}}\right)^x = 2^9$

$\left(2^{\frac{3}{2}}\right)^x = 2^9$

$2^{\frac{3}{2}x} = 2^9$

$\because$ Bases are equal so

$\frac{3}{2}x = 9$

$x = \frac{9 \times 2}{3}$

$x = \frac{\cancel{18}^{6}}{\cancel{3}}$

$x = 6$ **Ans**

Q.6 Find the value of x from the following statements.

(i) $\log_2 x = 5$

Solution: $\log_2 x = 5$

Write in exponential form.

$2^5 = x$

$32 = x$ **Ans**

(ii) $\log_{81} 9 = x$

Solution: $\log_{81} 9 = x$

Writing in the exponential form.

$81^x = 9$

$\left(9^2\right)^x = 9$

$9^{2x} = 9$

$2x = 1$

$x = \dfrac{1}{2}$ **Ans**

(iii) $\log_{64} 8 = \dfrac{x}{2}$

Solution: $\log_{64} 8 = \dfrac{x}{2}$

Writing in exponential form.

$64^{\frac{x}{2}} = 8$

$\left(8^{\cancel{2}}\right)^{\frac{x}{\cancel{2}}} = 8$

$8^x = 8$

$x = 1$ **Ans**

(iv) $\log_x 64 = 2$

Solution: $\log_x 64 = 2$

Writing in exponential form

$x^2 = 64$

$x^2 = 8^2$

$x = 8$ **Ans**

(v) $\log_3 x = 4$

Solution: $\log_3 x = 4$

$3^4 = x$

$81 = x$

Or $x = 81$ **Ans**

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Exercise 3.3

Q.1 Write the following into sum or difference $\log(A\times B)$

(i) $\log(A\times B)$

Solution: $\log(A\times B)$

$$\log A\times B=\log A+\log B \text{ **Ans**}$$

(ii) $\log\dfrac{15.2}{30.5}$

Solution: $\log\dfrac{15.2}{30.5}$

$$\log\frac{15.2}{30.5}=\log 15.2-\log 30.5 \quad \textbf{Ans}$$

(iii) $\log\dfrac{21\times 5}{8}$

Solution: $\log\dfrac{21\times 5}{8}$

$$\log\frac{21\times 5}{8}=\log(21\times 5)-\log 8$$
$$=\log 21+\log 5-\log 8 \quad \textbf{Ans}$$

(iv) $\log\sqrt[3]{\dfrac{7}{15}}$

Solution: $\log\sqrt[3]{\dfrac{7}{15}}$

$$\log\sqrt[3]{\frac{7}{15}}=\log\left(\frac{7}{15}\right)^{\frac{1}{3}}$$
$$=\frac{1}{3}\log\left(\frac{7}{15}\right)$$
$$=\frac{1}{3}(\log 7-\log 15)$$
$$=\frac{1}{3}\log 7-\frac{1}{3}\log 15 \quad \textbf{Ans}$$

(v) $\log\dfrac{(22)^{\frac{1}{3}}}{5^3}$

Solution: $\log\dfrac{(22)^{\frac{1}{3}}}{5^3}$

$$\log\frac{(22)^{\frac{1}{3}}}{5^3}=\log 22^{\frac{1}{3}}-\log 5^3$$
$$=\frac{1}{3}\log 22-3\log 5 \quad \textbf{Ans}$$

(vi) $\log\dfrac{25\times 97}{29}$

Solution: $\log\dfrac{25\times 97}{29}$

$$\log\frac{25\times 47}{29}=\log(25\times 47)-\log 29$$
$$=\log 25+\log 47-\log 29 \quad \textbf{Ans}$$

Q.2 Express $\log x-2\log x+3\log(x+1)-\log(x^2-1)$ **as a single logarithm.**

Solution:

$$\log x-2\log x+3\log(x+1)-\log(x^2-1)$$
$$=\log x-\log x^2+\log(x+1)^3-\log(x^2-1)$$
$$=\log\left(\frac{x}{x^2}\right)+\log\frac{(x+1)^3}{x^2-1}$$
$$=\log\left(\frac{x}{x^2}\times\frac{(x+1)^3}{x^2-1}\right)$$
$$=\log\left(\frac{x(x+1)^3}{x^2(x^2-1)}\right)$$
$$=\log\frac{\cancel{x}(x+1)^2\,\cancel{(x+1)}}{x\times\cancel{x}(x-1)\,\cancel{(x+1)}}$$
$$=\log\frac{(x+1)^2}{x(x-1)} \quad \textbf{Ans}$$

Q.3 Write the following in the form of a single logarithm.

(i) $\log 21+\log 5$

Solution: $\log 21+\log 5$

$=\log(21\times 5)$ **Ans**

(ii) $\log 25-2\log 3$

Solution: $\log 25-2\log 3$

$=\log 25-2\log 3$

$=\log 25-\log 3^2$

$=\log\dfrac{25}{3^2}$ **Ans**

(iii) $2\log x-3\log y$

Solution: $2\log x-3\log y$

$=2\log x-3\log y$

$=\log x^2-\log y^3$

$=\log\dfrac{x^2}{y^3}$ **Ans**

(iv) $\log 5+\log 6-\log 2$

Solution: $\log 5+\log 6-\log 2$

$=\log 5+\log 6-\log 2$

$=\log(5\times 6)-\log 2$

$=\log\dfrac{5\times 6}{2}$ **Ans**

Q.4 Calculate the following.

(i) $\log_3 2\times\log_2 81$

Solution: $\log_3 2\times\log_2 81$

$=\dfrac{\cancel{\log 2}}{\log 3}\times\dfrac{\log 81}{\cancel{\log 2}}$

$=\dfrac{\log 81}{\log 3}$

$=\dfrac{\log 3^4}{\log 3}$

$=\dfrac{4\cancel{\log 3}}{\cancel{\log 3}}$

$=4$ **Ans**

(ii) $\log_3\times\log_3 25$

Solution: $\log_3\times\log_3 25$

$=\dfrac{\cancel{\log 3}}{\log 5}\times\dfrac{\log 25}{\cancel{\log 3}}$

$=\dfrac{\log 25}{\log 5}$

$=\dfrac{\log 5^2}{\log 5}$

$=\dfrac{2\cancel{\log 5}}{\cancel{\log 5}}$

$=2$ **Ans**

Q.5 If $\log 2=0.3010, \log 3=0.4771$ **and** $\log 5=0.6990,$ **then find the values of the following.**

(i) $\log 32$

$=\log 2^5$

$\because$ using 3^{rd} law of logarithm

$=5\log 2$

By putting the value of $\log 2$

$=5(0.3010)$

$=1.5050$ **Ans**

(ii) log 24

Solution: log 24

$=\log(2^3\times 3)$

$=\log 2^3+\log 3$

$=3\log 2+\log 3$

By putting the value of $\log 2$ and $\log 3$

$=3(0.3010)+0.4771$

$=0.9030+0.4771$

$=1.3801$ **Ans**

(iii) $\log\sqrt{3\frac{1}{3}}$

Solution: $\log\sqrt{3\frac{1}{3}}$

$=\log\left(\dfrac{10}{3}\right)^{\frac{1}{2}}$

$=\frac{1}{2}\log\left[\frac{2\times5}{3}\right]$

$=\frac{1}{2}(\log2+\log5-\log3)$

By putting the values of $\log2,\log3$ and $\log5$

$=\frac{1}{2}(0.3010+0.69900-0.4771)$

$=\frac{1}{2}(1-0.4771)$

$=\frac{1}{2}(0.5229)$

$=0.26145$ **Ans**

(iv) $\log\frac{8}{3}$

Solution: $\log\frac{8}{3}$

$=\log\frac{2^3}{3}$

$=\log2^3-\log3$

$=3\log2-\log3$

By putting the values of $\log2$ and $\log3$

$=3(0.3010)-0.4771$

$=0.9030-0.4771$

$=0.4259$ **Ans**

(v) $\log30$

Solution: $\log30$

$=\log(5\times2\times3)$

$\because$ using first law of logarithm

$=\log5+\log2+\log3$

By putting the values of $\log2,\log3$ $\log5$

$=(0.6990)+(0.3010)+(0.4771)$

$=1.4771$ **Ans**

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Exercise 3.4

Q.1 Use log tables to find the value of

(i) 0.8176×13.64

Solution: 0.8176×13.64

Suppose

$x = 0.8176 \times 13.64$

Taking log on both sides

$\log x = \log(0.8176 \times 13.64)$

According to first law of logarithm

$\log x = \log 0.8176 + \log 13.64$

$= \bar{1}.9125 + \underline{1}.1348$

$\log x = -1 + 0.9125 + 1.1348$

$\log x = 1.0473$

To find antilog

$x = \text{antilog}\, 1.0473$

Ch = 1

$x = 1.115$

Reference point

$x = 11.15$ **Ans**

(ii) $(789.5)^{\frac{1}{8}}$

Solution: $(789.5)^{\frac{1}{8}}$

Let $x = (789.5)^{\frac{1}{8}}$

Taking log on both sides

$\log x = \log(789.5)^{\frac{1}{8}}$

According to third law

$\log x = \frac{1}{8}\log(789.5)$

$\log x = \frac{1}{8}(2.8974)$

$= \frac{2.8974}{8}$

$\log x = 0.3622$

To find antilog

x = antilog 0.3622

Characteristics = 0

$x = 2.302$

Reference point

$x = 2.302$ **Ans**

(iii) $\frac{0.678 \times 9.01}{0.0234}$

Solution: $\frac{0.678 \times 9.01}{0.0234}$

Suppose

$x = \frac{0.678 \times 9.01}{0.0234}$

Taking log on both sides

$\log x = \log \frac{0.678 \times 9.01}{0.0234}$

According to 1st and 2nd law of log

$\log x = \log 0.678 + \log 9.01 - \log 0.0234$

$\log x = \bar{1}.8312 + 0.9547 - \bar{2}.3692$

$= -1 + 0.8312 + 0.9547 - (-2 + 0.3692)$

$= 2.4167$

To find antilog

x = antilog 2.4167

Characteristics = 2

$x = 2.610$

$x = 261.0$ **Ans**

(iv) $\sqrt[5]{2.709} \times \sqrt[7]{1.239}$

Solution: $\sqrt[5]{2.709} \times \sqrt[7]{1.239}$

$(2.709)^{\frac{1}{5}} \times (1.239)^{\frac{1}{7}}$

Suppose:

$x = (2.709)^{\frac{1}{5}} \times (1.239)^{\frac{1}{7}}$

Taking log on both side

$\log x = \log\left[(2.709)^{\frac{1}{5}} \times (1.239)^{\frac{1}{7}}\right]$

According to law of logarithm

$\log x = \log(2.709)^{\frac{1}{5}} + \log(1.239)^{\frac{1}{7}}$

According to third law of logarithm

$\log x = \frac{1}{5}\log(2.709) + \frac{1}{7}\log(1.239)$

$\log x = \frac{1}{5}\log(2.709) + \frac{1}{7}\log(1.239)$

$=\frac{1}{5}0.4328+\frac{1}{7}0.0931$

$=\frac{0.4328}{5}+\frac{0.0931}{7}$

$0.0866+0.0133$

$=0.0999$

To find antilog

x = antilog 0.999

Characteristics = 0

$x=1.259$

Reference point

$x=1.259$ **Ans**

(v) $\frac{1.23\times 0.6975}{0.0075\times 1278}$

Solution: $\frac{1.23\times 0.6975}{0.0075\times 1278}$

Suppose

$x=\frac{1.23\times 0.6975}{0.0075\times 1278}$

$\log x=\log\frac{1.23\times 0.6975}{0.0075\times 1278}$

$=\log(1.23\times 0.6975)-\log(0.0075\times 1278)$

$=\log 1.23+\log 0.6975-(\log 0.0075+\log 1278)$

$=\log 1.23+\log 0.6975-\log 0.0075-\log 1278$

$=0.0899+\bar{1}.8435-\bar{3}.8751-3.1065$

$=0.8999+(-1+0.8435)-(-3+0.8751)+3.1065$

$=-1.0482$

$\log x=-2+2-1.0482$

$\log x=02+0.9515$

$\log x=\bar{2}.9518$

To find antilog

x = antilog $\bar{2}.9518$

Ch = $\bar{2}$

$x=8950$

$=0.08950$ **Ans**

(vi) $\sqrt[3]{\frac{0.7214\times 20.37}{60.8}}$

Solution: $\sqrt[3]{\frac{0.7214\times 20.37}{60.8}}$

Let $x=\left[\frac{0.7214\times 20.37}{60.8}\right]^{\frac{1}{3}}$

Taking log on both sides

$\log x=\log\left(\frac{0.7214\times 20.37}{60.8}\right)^{\frac{1}{3}}$

3rd of logarithm

$\log x=\frac{1}{3}\log\left[\frac{0.7214\times 20.37}{60.8}\right]$

According to first and 2nd law

$\log x=\frac{1}{3}[\log 0.7214+\log 37-\log 60.8]$

$\log x=\frac{1}{3}[\bar{1}.8582+1.3089-1.7839]$

$\frac{1}{3}[-1+0.8582+1.3089-1.7839]$

$=\frac{1}{3}(-0.6168)$

$=-0.2056$

$\log x$ is in negative, so

$\log x=-1+1-0.2056$

$=-1+79144$

$=\bar{1}.7944$

To find antilog

x = antilog $\bar{1}.7944$

Ch = $\bar{1}$

$x=6229$

Reference point

0.6229 **Ans**

(vii) $\frac{83\times\sqrt[3]{92}}{127\times\sqrt[5]{246}}$

Solution: $\frac{83\times\sqrt[3]{92}}{127\times\sqrt[5]{246}}$

Suppose: $x=\frac{83\times\sqrt[3]{92}}{127\times\sqrt[5]{246}}$

$$x=\frac{83\times(92)^{\frac{1}{3}}}{127\times(246)^{\frac{1}{5}}}$$

Taking on both side

$$\log x=\log\frac{83\times(92)^{\frac{1}{3}}}{127\times(246)^{\frac{1}{5}}}$$

According to 1st and 2nd law of log

$$\log x=\log 83+\log(92)^{\frac{1}{3}}-\log 127-\log(246)^{\frac{1}{5}}$$

According to third law of log

$$\log x=\log 83+\frac{1}{3}\log 92-\log 27-\frac{1}{5}\log 246$$

$$\log x=(1.9191)+\frac{1}{3}(1.9638)-(2.1038)$$

$$-\frac{1}{5}(2.3909)$$

$= 1.9191 + 0.65460 - 2.1038 - 0.47818$

$=1.9191+0.6546-2.1038-0.47818$

$=-0.0083$

$\log x$ is in negative, so

$\log x=-1+1-0.0083$

$=-1+0.9917$

$=\bar{1}.9917$

To find antilog

$x=$ antilog $\bar{1}.9917$

Ch = $\bar{1}$

$x=9.811$

Reference point

$x=0.9811$ **Ans**

(viii) $\dfrac{(438)^3\sqrt{0.056}}{(388)^4}$

Solution: $\dfrac{(438)^3\sqrt{0.056}}{(388)^4}$

Suppose: $x=\dfrac{(438)^3\sqrt{0.056}}{(388)^4}$

$$x=\frac{(438)^3(0.056)^{\frac{1}{2}}}{(388)^4}$$

$$x=\frac{(438)^3(0.056)^{\frac{1}{2}}}{(388)^4}$$

Taking log on both side

$$\log x=\log\left(\frac{(438)^3(0.056)^{\frac{1}{2}}}{(388)^4}\right)$$

According to 1st and 2nd law

$$\log x=\log(438)^3+\log(0.056)^{\frac{1}{2}}-\log(388)^4$$

According to third law

$$\log x=3\log(438)+\frac{1}{2}\log(0.056)-4\log(38)$$

$$\log x=3(2.6415)+\frac{1}{2}(\bar{2}.7482)-4(2.5888)$$

$$=7.9245+\frac{1}{2}(-2+0.7482)-10.3552$$

$$=7.9245+\frac{1}{2}(-1.2518)-10.3552$$

$=7.9245-0.6259-10.3552$

$=-3.0566$

log is in negative, so

$\log x=-4+4-3.0566$

$=-4+0.9434$

To find antilog

$x=$antilog $\bar{4}.9434$

Ch = $\bar{4}$

$x=8778$

Reference point

$=0.0008778$ **Ans**

Q.2 A gas is expanding according to the law $pv^n = C$. Find C when p = 80, v = 3.1 and $n=\frac{5}{4}$.

Solution: Given that $pv^n = C$

Taking log on both sides

Log (pv^n)= log C

Log P+log v^n= log C

Log C =log P+log v^n

Log C =log P+n log v

Putting P=80, v=3.1 and $n=\frac{5}{4}$

Log C = log 80+$\frac{5}{4}$ log 3.1
=1.9031+$\frac{5}{4}$(0.4914)
=1.9031+0.6143
Log C=2.5174
Taking antilog both sides
C=Antilog (2.5174)
C=329.2 **Ans:**

Q.3 The formula p= 90 $(5)^{-q/10}$ applies to the demand of a product, where q is the number of units and p is the price of one unit. How many units will be demanded if the price is Rs 18.00?

Solution: Given that $p = 90(5)^{\frac{-q}{10}}$
Taking log on both sides
$Log\ p = \log\left(90(5)^{\frac{-q}{10}}\right)$
$Log\ p = \log 90 + \log 5^{\frac{-q}{10}}$
Log P =log 90- $\frac{q}{10}$ log 5
Log 18= log 90- $\frac{q}{10}$ log 5
(P =18)
1.2553=1.9542- $\frac{q}{10}$ x 0.6990
1.2553-1.9542 =- $\frac{q}{10}$ x 0.6990
-0.6989 x 10 = - q x 0.6990
-6.989=-q x 0.6996
6.989=q x 0.6996
$\frac{6.989}{0.6990}$ =q
q =10 approximately
Hence 10 units will be demanded

Q.4 If $A = \pi r^2$, **find A, when** $\pi = \frac{22}{7}$ **and** $r = 15$ **.**

Solution: Given that $A = \pi r^2$
Taking log on both sides
Log A=log πr^2
Log A=log π +log r^2
Log A=log π +2 log r
Putting $\pi = \frac{22}{7}$ and r =15
Log A=log $\frac{22}{7}$ +2 log 15
=log 22-log 7+2 log 15
=1.3424-0.8451+2(1.1761)
=0.4973+2.3522
Log A=2.8495
Taking antilog on both sides
A=antilog 2.8495
A=707.1 **Ans**

Q.5 If $V = \frac{1}{3}\pi r^2 h$, **find V, when** $\pi = \frac{22}{7}$, $r = 2.5$ **and** h**=4.2.**

Solution: Given that $V = \frac{1}{3}\pi r^2 h$
Taking log on both sides
$\log V = \log \frac{1}{3}\pi r^2 h$
=log$\frac{1}{3}$ +log$\pi r^2 h$
=log 1-log 3+ log πr^2 + log h
= 0-0.4771+ log π +log r^2+ log h
=-0.4771+ log $\frac{22}{7}$ +2 log r + log h
$\left(\pi = \frac{22}{7}, r = 2.5 \text{ and } h = 4.2\right)$
$= -0.4771 + \log 22 - \log 7 + 2\log 2.5 + \log 4.2$
$= -0.4771 + 1.3424 - 0.8450 + 2x0.3979 + 0.6232$
$= -0.4771 + 1.3424 - 0.8450 + 0.7959 + 0.6232$
Log V=1.4394
Taking antilog on both sides
V=antilog 1.4394
V=27.50 **Ans**

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❖ اگر کسی کو بھی گروپ کے متعلق کسی قسم کی شکایت یا تجویز کی صورت میں ایڈمن سے رابطہ کیجئے۔

❖ سب سے اہم بات:

گروپ میں کسی بھی قادیانی، مرزائی، احمدی، گستاخِ رسول، گستاخِ امہات المؤمنین، گستاخِ صحابہ و خلفائے راشدین حضرت ابو بکر صدیق، حضرت عمر فاروق، حضرت عثمان غنی، حضرت علی المرتضیٰ، حضرت حسنین کریمین رضوان اللہ تعالیٰ اجمعین، گستاخِ اہلبیت یا ایسے غیر مسلم جو اسلام اور پاکستان کے خلاف پراپیگنڈا میں مصروف ہیں یا ان کے روحانی و ذہنی سپورٹرز کے لئے کوئی گنجائش نہیں ہے لہذا ایسے اشخاص بالکل بھی گروپ جوائن کرنے کی زحمت نہ کریں۔ معلوم ہونے پر فوراً ریموو کر دیا جائے گا۔

❖ تمام کتب انٹرنیٹ سے تلاش / ڈاؤنلوڈ کر کے **فری آف کاسٹ** وٹس ایپ گروپ میں شیئر کی جاتی ہیں۔ جو کتاب نہیں ملتی اس کے لئے معذرت کر لی جاتی ہے۔ جس میں محنت بھی صَرف ہوتی ہے لیکن ہمیں آپ سے صرف دعاؤں کی درخواست ہے۔

❖ [redacted]

لیڈیز کے لئے الگ گروپ کی سہولت موجود ہے جس کے لئے ویریفکیشن ضروری ہے۔

❖ [redacted] رابطہ کریں اور جواب کا انتظار فرمائیں۔ برائے مہربانی اخلاقیات کا خیال رکھتے ہوئے موبائل پر کال یا ایم ایس کرنے کی کوشش ہر گز نہ کریں۔ ورنہ گروپس سے تو ریموو کیا ہی جائے گا بلاک بھی کیا جائے گا۔

[redacted]

0306-7163117	0343-7008883	0333-8033313
محمد سلمان سلیم	پاکستان زندہ باد	راؤ اماز

[redacted]

Exercise 1.4

Q1. Which of the following product of matrices is conformable for multiplication?

(i) $\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$ **(iv)** $\begin{bmatrix} 1 & 2 \\ 0 & -1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$

(v) $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ -2 & 3 \end{bmatrix}$

Solution:

Two matrices are conformable for multiplication if the numbers of columns of first matrix are equal to number of rows of second matrix.

So, according to this definition:

(i) is conformable for multiplication (because the first matrix has two columns and second matrix has same number of rows).

(ii) is conformable for multiplication (because the first matrix has two columns and second matrix has same number of rows).

(iii) is not conformable for multiplication (because the first matrix has just one column and second matrix has two rows).

(iv) is conformable for multiplication (because the first matrix has just two columns and second matrix has the same number of rows).

(v) is conformable for multiplication (because the first matrix has three columns and second matrix has same number of rows).

Q2. **If A =** $\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$, **B =** $\begin{bmatrix} 6 \\ 5 \end{bmatrix}$,

find (i) AB (ii) BA (if possible)

Solution:

(i) AB

$$= \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times6+0\times5 \\ (-1)\times6+2\times5 \end{bmatrix}$$

$$= \begin{bmatrix} 18+0 \\ -6+10 \end{bmatrix}$$

$$= \begin{bmatrix} 18 \\ 4 \end{bmatrix}$$

So, AB $= \begin{bmatrix} 18 \\ 4 \end{bmatrix}$

(ii) BA

BA is not possible (because number of columns of B is

Not equal to number of rows of A)

Q3. **Find the following products**

(i) $\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ -4 \end{bmatrix}$

(iii) $\begin{bmatrix}-3 & 0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$ **(iv)** $\begin{bmatrix}6 & -0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$

(v) $\begin{bmatrix}1 & 2\\-3 & 0\\6 & -1\end{bmatrix}\begin{bmatrix}4 & 5\\0 & -4\end{bmatrix}$ **(vi)** $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$

Solution:

(i) $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$

$= \begin{bmatrix}1\times4+2\times0\end{bmatrix}$

$= \begin{bmatrix}4+0\end{bmatrix}$

$= \begin{bmatrix}4\end{bmatrix}$

So, $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix} = \begin{bmatrix}4\end{bmatrix}$

(ii) $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}5\\-4\end{bmatrix}$

$= \begin{bmatrix}1\times5+2\times(-4)\end{bmatrix}$

$= \begin{bmatrix}5-8\end{bmatrix}$

$= \begin{bmatrix}-3\end{bmatrix}$

So, $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}5\\-4\end{bmatrix} = \begin{bmatrix}-3\end{bmatrix}$

(iii) $\begin{bmatrix}-3 & 0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$

$$= \left[(-3)\times 4+0\times 0\right]$$

$$= \left[-12\right]$$

So, $\begin{bmatrix}-3 & 0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}=\left[-12\right]$

(iv) $\begin{bmatrix}6 & -0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$

$$= \left[6\times 4+0\times 0\right]$$

$$= \left[24+0\right]$$

$$= \left[24\right]$$

So, $\begin{bmatrix}6 & -0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix} = \left[24\right]$

(v) $\begin{bmatrix}1 & 2\\-3 & 0\\6 & -1\end{bmatrix}\begin{bmatrix}4 & 5\\0 & -4\end{bmatrix}$

$$= \begin{bmatrix}1\times 4+2\times 0 & 1\times 5+2\times(-4)\\-3\times 4+0\times 0 & -3\times 5+0\times(-4)\\6\times 4+(-1)\times 0 & 6\times 5+(-1)\times(-4)\end{bmatrix}$$

$$= \begin{bmatrix}4+0 & 5+(-8)\\-12+0 & -15+0\\24+0 & 30+4\end{bmatrix}$$

$$= \begin{bmatrix}4 & -3\\-12 & -15\\24 & 34\end{bmatrix}$$

Exercise 1.5

Q1. Find the determinant of the following matrices.

(i) **A =** $\begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$ **(ii)** **B =** $\begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$

(iii) **C =** $\begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$ **(iv)** **D =** $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

Solution:

(i) **A =** $\begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$

Determinant of matrix Ais calculated as:

$|\mathbf{A}|$ = det A = $\begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$ = (-1) x 0 - 2 x 1

$|\mathbf{A}|$ **= 0 – 2 = – 2**

(ii) **B =** $\begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$

Determinant of matrix B is calculated as:

$|\mathrm{B}|$ = det B = $\begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$ = 1 x (-2) - 3 x 2

$|\mathbf{B}|$ **= – 2 – 6 = – 8**

(iii) **C =** $\begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$

Determinant of matrix C is calculated as:

$|C|$ = det C = $\begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$ = 3 x 2 - 2 x 3

$|C|$ **= 6 – 6 = 0**

(iv) **D =** $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

Determinant of matrix D is calculated as:

$|D|$ = det D = $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ = 3 x 4 - 2 x 1

$|D|$ **= 12 – 2 = 10**

Q2. Find which of the following matrices are singular or non-singular?

(i) **A =** $\begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$ **(ii)** **B =** $\begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$

(iii) **C =** $\begin{bmatrix} 7 & -9 \\ 3 & 5 \end{bmatrix}$ **(iv)** **D =** $\begin{bmatrix} 5 & -10 \\ -2 & 4 \end{bmatrix}$

Solution:

A matric is said to be singular if its determinant is equal to zero. i.e., $|A| = 0$

(i) **A =** $\begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$

Determinant of matrix A is calculated as:

$|A|$ = det A = $\begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$ = 3 x 4 - 2 x 6

$|\mathbf{A}|$ **= 12 – 12 = 0**

As, determinant of A is equal to zero so, A is a singular matrix.

(ii) **B =** $\begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$

Determinant of matrix B is calculated as:

$|B|$ = det B = $\begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$ = 4 x 2 - 1x 3

$|\mathbf{B}|$ **= 8 - 3 = 5 $\neq$ 0**

As, determinant of B is not equal to zero so, B is a not a singular matrix.

(iii) **C =** $\begin{bmatrix} 7 & -9 \\ 3 & 5 \end{bmatrix}$

Determinant of matrix C is calculated as:

$|C|$ = det C = $\begin{bmatrix} 7 & -9 \\ 3 & 5 \end{bmatrix}$ = 7 x 5 – 3 x (-9)

$|\mathbf{C}|$ **= 35 + 27 = 62 $\neq$ 0**

As, determinant of C is not equal to zero so, C is a not a singular matrix.

(iv) **D =** $\begin{bmatrix} 5 & -10 \\ -2 & 4 \end{bmatrix}$

Determinant of matrix D is calculated as:

$|D|$ = det D = $\begin{bmatrix} 5 & -10 \\ -2 & 4 \end{bmatrix}$ = 5 x 4 – (-10) x (-2)

$|\mathbf{D}|$ **= 20 – 20 = 0**

Exercise 1.6

Q1. Use matrices, if possible, to solve the following systems of linear equations by:

(i) the matrix inversion method

(ii) the Cramer's rule

(i) $2x - 2y = 4$, $3x + 2y = 6$ **(ii)** $2x + y = 3$, $6x + 5y = 1$

(iii) $4x + 2y = 8$, $3x - y = -1$ **(iv)** $3x - 2y = -6$, $5x - 2y = -10$

(v) $3x - 2y = 4$, $-6x + 4y = 7$ **(vi)** $4x + y = 9$, $-3x - y = -5$

(vii) $2x - 2y = 4$, $-5x - 2y = -10$ **(viii)** $3x - 4y = 4$, $x + 2y = 8$

(i) Solution by Matrix Inversion Method:

(i) $2x - 2y = 4$

$3x + 2y = 6$

Step 1

$$\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

Step 2

The coefficient matrix M = $\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$ is non-singular because

det M = $\begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix}$ = $2\times2-3\times(-2)$ = 4 + 6 = 10 $\neq$ 0

Step 3

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1}\begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|M|} Adj\,M \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 2\times4+2\times6 \\ -3\times4+2\times6 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 8+12 \\ -12+12 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 20 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$\therefore \quad x=2\ ,\ \ y=0$

(ii) 2x + y = 3

6x + 5y = 1

Step 1

$$\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Step 2

The coefficient matrix M = $\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}$ is non-singular because

det M = $\begin{vmatrix} 2 & 1 \\ 6 & 5 \end{vmatrix}$ = $2\times5-1\times6$ = 10 - 6 = 4 $\neq$ 0

Step 3

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|M|} Adj\, M \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 5\times 3+(-1)\times 1 \\ -6\times 3+2\times 1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 14 \\ -16 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{7}{2} \\ -4 \end{bmatrix}$$

$\therefore \quad x = \frac{7}{2}\ ,\ y = -4$

(iii) 4x + 2y = 8

3x – y = – 1

Step 1

$$\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

Step 2

The coefficient matrix M = $\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$ is non-singular because

det M = $\begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix}$ = $4\times(-1)-2\times 3$ = -4 - 6 = - 10 $\neq$ 0

Step 3

Review Exercise

Q1. Select the correct answer in each of the following.

(i) The order of matrix $\begin{bmatrix} 2 & 1 \end{bmatrix}$ is …………

(a) 2-by-1 (b) 1-by-2

(c) 1-by-1 (d) 2-by-2

(ii) $\begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$ is called ………… matrix.

(a) zero (b) unit

(b) scalar (d) singular

(iii) Which is order of square matrix…………

(a) 2-by-2 (b) 1-by-2

(c) 2-by-1 (d) 3-by-2

(iv) Order of transpose of $\begin{bmatrix} 2 & 1 \\ 0 & 1 \\ 3 & 2 \end{bmatrix}$ …………

(a) 3-by-2 (b) 2-by-3

(c) 1-by-3 (d) 3-by-1

(v) Adjoint of $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$ is …………

(a) $\begin{bmatrix} -1 & -2 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 2 \\ 0 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$

(vi) Product of $\begin{bmatrix} x & y \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ is............

(a) $[2x+y]$ (b) $[x-2y]$

c) $[2x-y]$ (d) $[x+2y]$

(vii) If $\begin{vmatrix} 2 & 6 \\ 3 & x \end{vmatrix}$ = 0, then x is equal to…a = ...

(a) 9 (b) - 6

(c) 6 (d) - 9

(viii) If $X+\begin{bmatrix} -1 & -2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then X is equal to............

(a) $\begin{bmatrix} 2 & 2 \\ 2 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 2 \\ 2 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$

<u>Answers</u>

(i) b	(ii) c	(iii)a	(iv) b
(v) a	(vi) c	(vii) a	(viii) d

Q2. Complete the following:

(i) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is called ……….. matrix.

(ii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is called ……….. matrix.

(iii) Additive inverse of $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$ is ………..

(iv) In matrix multiplication, in general, AB ……….. BA.

(v) Matrix A + B may be found if order of A and B is ………..

(vi) A matrix is called ………. matrix if number of rows and columns are equal.

Answers:

(i) Null **(ii) Unit** **(iii)** $\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$

(iv) $\neq$ **(v) Same** **(vi) Square**

Q3. **If** $\begin{bmatrix} a+3 & 4 \\ 6 & b-1 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$**, then find a and b.**

Solution:

By comparing the corresponding elements, we get

$a+3=-3$

$a=-3-3=-6$

$a=-6$ Answer

and $b-1=2$

$b=2+1$